

Lund 1980

REVERSE OSMOSIS, ULTRAFILTRATION
AND MASS TRANSFER IN TURBULENT
DUCT FLOW

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**LUND INSTITUTE OF TECHNOLOGY
LUND - SWEDEN**



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AVHANDLINGEN KOMMER ATT FÖRSVARAS VID EN OFFENTLIG
DISPUTATION I SAL B, KEMICENTRUM, LUND, LÖRDAGEN
DEN 13 DECEMBER 1980, KL 1015.

Coden: LUTKDH/(TKKA-1002)/1-18/(1980)

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DISSERTATION
DECEMBER 1980



Organization LUND UNIVERSITY Chemical Engineering Chemical Centre P.O.Box 740 S-220 07 LUND 7, SWEDEN		Document name DOCTORAL DISSERTATION			
		Date of issue November 1980			
		CODEN: LUTKDH/(TKKA-1002)/1-18/(1980)			
Author(s) Peter Eriksson		Sponsoring organization			
Title and subtitle Reverse Osmosis, Ultrafiltration and Mass Transfer in Turbulent Duct Flow					
Abstract At Lund Institute of Technology a test plant has been built to perform experimental investigations of possible applications of reverse osmosis and ultrafiltration. Some results are presented for ultrafiltration of effluents from pulp bleaching plants and dyehouses, and concentration of effluents from polyalcohol production by reverse osmosis. In 1977 an inventory of reverse osmosis plants for <u>cheese whey</u> concentration was made, and their operation and <u>economy</u> are described. An investigation has been carried out on ultrafiltration of <u>spent sulfite liquor</u> . In ultrafiltration the <u>permeate flux</u> is very much dependent on the mass transfer coefficient. A critical <u>examination</u> of published correlations for the <u>mass transfer coefficient</u> at fully developed turbulent flow in circular ducts with <u>smooth nonpermeable walls</u> and constant physical properties at $Sc > 0.5$ is made. For the high Schmidt numbers encountered in <u>reverse osmosis and ultrafiltration</u> ($Sc > 500$), it is investigated for turbulent flow how the mass transfer coefficient depends on <u>rough walls</u> , the <u>entrance region</u> , turbulent Reynolds numbers below 10^4 , <u>noncylindrical geometry</u> , <u>variable physical properties</u> and <u>permeable walls</u> . The solute rejection of the membrane decreases with increasing pressure difference if the permeate flux is greater than the zero flux limiting mass transfer coefficient. In ultrafiltration at turbulent flow the energy consumption per unit volume permeate is only very slightly dependent on the hydraulic diameter of the duct.					
Key words					
Classification system and/or index terms (if any)					
Supplementary bibliographical information			Language English		
ISSN and key title			ISBN		
Recipient's notes	Number of pages 20		Price		
	Security classification				

Distribution by (name and address)
Dept. of Chemical Engineering, Chemical Centre, P.O.Box 740, 220 07 Lund 7, Sweden.

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REVERSE OSMOSIS, ULTRAFILTRATION AND MASS TRANSFER IN TURBULENT DUCT FLOW

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This thesis consists of this summary and of the following papers, later referred to by Roman numerals:

- I. Eriksson, G., Eriksson, P., Hallström, B. and Wimmerstedt, R.
Membrane Technology Research at Lund Institute of Technology.
Desalination, 27, 81-89 (1978)
- II. Eriksson, P.
Concentration of Whey by Reverse Osmosis-Inventory and Operating Experiences.
Nordeuropeisk Mejeri-Tidskrift, 43, 238-46 (1977)
- III. Eriksson, P.
Ultrafiltration for Recovery of Lignosulfonates from Spent Sulfite Liquor.
AIChE Symp. Ser. No. 197, 76, 316-20 (1980)
- IV. Eriksson, P.
Mass Transfer in Turbulent Duct Flow.
Report LUTKDH/(TKKA-3006)/1-119/(1980), Dept. of Chem. Eng., Lund Institute of Technology, 1980

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INTRODUCTION

Reverse osmosis and ultrafiltration are liquid filtration processes where even dissolved molecules can be rejected. The filtering medium is a thin porous asymmetric film, which is called a membrane. The pore size is very small in a thin layer at the membrane surface facing the processed liquid. In ultrafiltration the pore width varies from slightly less than 1 nm to about 10 nm, so convective flow prevails in the pores and molecules are rejected mainly because of their size. In reverse osmosis the pores are so small, that most of the permeating molecules are in contact with the pore wall. Then there is no convective flow. In this type of transport (diffusion) the solute rejection properties of the membrane are not only dependent on the size of the solute molecules but also on chemical and electrical forces between solute, solvent and membrane.

Reverse osmosis and ultrafiltration began to be used on industrial scale at the end of the 1960's, and by the beginning of the 1970's they were established processes. The main use of reverse osmosis is for water desalination, but there are many other industrial applications, for example concentration of cheese whey and milk in the dairy industry. Ultrafiltration is much used for producing rinse water in the electro-coating process, for fractionation of oil/water emulsions and in the dairy industry for fractionation of cheese whey and milk.

MEMBRANE TECHNOLOGY RESEARCH AT LUND INSTITUTE OF TECHNOLOGY

In order to promote industrial applications of reverse osmosis and ultrafiltration, the governmental Swedish Board for Technical Development financed the building of a "membrane station" at Lund Institute of Technology. At the membrane station are found both laboratory equipment and flexible pilot plants, which work with most of the membrane modules on the market. Here it is experimentally investigated, if it is technically and economically possible to use reverse osmosis and ultrafiltration for different applications. Some of these investigations are reported in paper I.

At present, membrane technology research is financed by a foundation consisting of 11 industries and the Swedish Board for Technical Development on an fifty-fifty basis.

CONCENTRATION OF CHEESE WHEY BY REVERSE OSMOSIS

Paper II considers concentration of cheese whey by reverse osmosis. At the beginning of 1977 about 15 such plants were in operation. All of them use plate and frame modules or tubular modules with about 12 mm inner diameter. The system design is either batch-wise or continuous. The continuous systems are either single pass or recirculating multistage. In the single pass plants the residence time of the whey is short, so a high temperature can be used without too much bacterial growth. This is an advantage as the permeate flux increases and the cooling costs decrease with increasing temperature. The upper limit of the temperature used is about 30°C, as a higher temperature will considerably shorten the membrane lifetime. With batch-wise and continuous recirculating multistage plants it is possible to achieve a high concentration ratio, 75% of the water can be removed compared to about 50% with a single pass plant. Plants with 50% water removal are dimensioned for an average permeate flux of 20-25 l/m²h at a pressure of 4.0 MPa, but of five observed plants three had a permeate flux of between 13 and 17 l/m²h.

Economically, capital costs predominate since they constitute more than 50% of the total calculated costs.

ULTRAFILTRATION OF SPENT SULFITE LIQUOR

Paper III considers ultrafiltration of spent sulfite liquor. In the manufacture of cellulose fiber pulps by the acid sulfite process, about 50% of the wood raw material remains in the spent sulfite liquor. This organic material should be a valuable raw product, but for a long time it was discarded, and caused a serious water pollution problem. At present most modern sulfite pulp mills evaporate and burn the spent sulfite liquor to produce steam and to recover the chemicals for reuse in the cooking operation. However, there are other more advanced applications where the organic material has a higher economic value. For example purified lignosulfonates can be used as dispersants and adhesives. This purified lignosulfonate fraction is obtained from ultrafiltration of spent sulfite liquor.

At present there are two commercial ultrafiltration plants in the world for purifying the lignosulfonate fraction in the spent sulfite liquor, and both use plate and frame modules. It is also possible to use tubular modules and perhaps also internally pressurized hollow fibers, but spiral wound modules and externally pressurized hollow fine fibre modules are not suitable for this application, because they will be clogged.

The temperature of the spent sulfite liquor is about 90°C , and as the permeate flux increases with increasing temperature, it is important that the membrane can withstand high temperatures. At present polysulfone membranes are used which withstand temperatures of up to 80°C .

The lignosulfonates will form a dynamic layer on the membrane. Since the lignosulfonates are strongly hydrophilic, their water permeability is fairly high. This means that the dynamic layer would not directly decrease the permeate flux to any considerable extent. However, this layer greatly reduces the back diffusion of molecules rejected by the membrane. If there are other slightly hydrophilic molecules which are rejected by the membrane, then the dynamic layer will cause an increased concentration of these molecules near the membrane surface. This will strongly decrease the permeate flux. Thus to get a high permeate flux in fractionation of spent sulfite liquor, the membrane must be coarse enough to let molecules not strongly hydrophilic through.

It is not possible to obtain a lignosulfonate fraction of desired purity simply by straightforward ultrafiltration, because the permeate flux decreases with increasing concentration ratio and will be zero before desired lignosulfonate purity is obtained. This is overcome by diafiltration, which means that water is added to the concentrate in one or a few stages in the ultrafiltration plant.

The costs for diafiltration are almost half the total costs for the ultrafiltration plant, and like the concentration of cheese whey by reverse osmosis, the capital costs constitute more than 50% of the total calculated costs.

MASS TRANSFER TO THE MEMBRANE SURFACE

As mentioned above it is the capital costs which dominate the economy for many reverse osmosis and ultrafiltration plants. The capital costs are almost directly proportional to the membrane area, so to achieve a good economy the membrane area should be small and thus the permeate flux high. The permeate flux depends on the resistance in the liquid phase, the resistance of a possible fouling layer on the membrane surface and the resistance of the membrane. The resistance in the liquid phase is characterized by the mass transfer coefficient, and paper IV, which is the main part of this thesis, deals with its determination for turbulent flow. Considered here first is the mass transfer coefficient for fully developed turbulent flow in circular ducts with smooth nonpermeable walls and constant physical properties.

Reynolds at the end of the 19:th century was the first to realize the connection between mass or heat transfer and momentum transfer. He assumed that in turbulent flow the molecular diffusion of mass and momentum were negligible compared to the mass and momentum transfer by whirls. This assumption is not valid in the vicinity of the wall where the whirl frequency is very low. There, the molecular diffusivity is important for the mass and momentum transfer. Thus the result of Reynolds' analysis overpredicts the mass transfer coefficient at low mass diffusivities, i.e. high Schmidt numbers.

Nernst (1904) tried to overcome the weakness of Reynolds' analysis by assuming that near the wall there was a thin stagnant film where mass and momentum transfer occurred only by molecular diffusion. Outside this film complete mixing was assumed to occur. These assumptions result in a correlation whereby the mass transfer coefficient is directly proportional to the mass diffusivity. However, there is always a small turbulent mass transport, even very close to the wall, which is of great importance when the mass diffusivity and hence the molecular mass transport are low. Because Nernst disregards the turbulent mass transport his correlation underpredicts the mass transfer coefficient at low mass diffusivities.

Other authors have combined the analyses of Reynolds and Nernst, assuming a thin film near the wall, where the turbulent transport is disregarded. Outside this film the molecular transport is disregarded. Some of the authors also include an intermediate region where both molecular and turbulent transport are considered. However, all these models, which assume a film near the wall where no turbulent transport occurs, underpredict the mass transfer coefficient at low molecular mass diffusivities. This is because the lower the molecular diffusivity the more important is the small turbulent transport near the wall, which has been disregarded.

The eddy diffusivity models define an eddy diffusivity according to Eq. (1).

$$j = (D + \epsilon_q) \frac{dc}{dy} \quad (1)$$

where j is the time average solute flux, D is the solute-solvent mass diffusivity, ϵ_q is the eddy diffusivity for mass, c is the time average solute concentration and y is the distance from the wall. Thus the turbulent transport towards the wall is represented by the term $\epsilon_q \cdot \frac{dc}{dy}$. However, the eddy diffusivity is not a physical property but merely a definition. Consequently the only way to determine ϵ_q is by measuring the solute flux towards the wall at known solute concentration at the wall and in the bulk, and then to evaluate ϵ_q from Eq. (1). The mass transfer coefficient can then be evaluated from ϵ_q and D . This is a roundabout method, since from experimental values of the wall solute flux (j_w) and solute concentrations at the wall (c_w) and in the bulk (c_b), the mass transfer coefficient can be evaluated directly from Eq (2).

$$j_w = k(c_b - c_w) \quad (2)$$

It is possible to determine ϵ_q from momentum transfer data, by assuming that the eddy diffusivities of mass and momentum are equal. However, it is not possible to evaluate the eddy diffusivity of momentum in a region very close to the wall, where it is negligible compared to the momentum diffusivity. For mass transfer in liquids the mass diffusivity is so low that the whole concentration boundary layer is within this region, so there is no use for the correlation of the eddy diffusivity of momentum.

According to the penetration theory, whirls penetrate to the wall or a short distance from the wall and then unsteady mass transfer occurs between the whirl and the surroundings. In the most simple penetration model (Higbie (1935)) all whirls reach the wall and stay there for a constant time. The resulting correlation for the mass transfer coefficient is poor. Harriott (1962) assumes that the whirl penetration depth and contact time vary according to selected distribution curves, and obtained fairly good results for the mass transfer coefficient. Ruckenstein (1958 and 1967) assumes that the whirls penetrate all the way to the wall and there develop a stationary boundary layer flow. The resulting mass transfer coefficients agree well with experimental data at low mass diffusivities.

Hughmark in a series of papers (1969, 1971 a, 1971 b, 1972, 1973, 1975 and 1977) derives a correlation for the mass transfer coefficient (Eq. (3)) which agrees very well with experimental data at all Schmidt numbers above one.

$$\frac{1}{k} = \frac{1}{\frac{u^*}{1.6 \cdot Sc} + 0.047 \cdot u^* \cdot Sc^{-\frac{2}{3}}} + \frac{1}{\frac{u^*}{33 \cdot Sc} + 0.062 \cdot u^* \cdot Sc^{-\frac{2}{3}}} + \frac{1}{7.16 \frac{D}{d_n} + f \cdot u_b} \quad (3)$$

where u^* is the friction velocity ($\sqrt{\tau_w/\rho}$), Sc is the Schmidt number, d_n is the hydraulic diameter, f is the fanning friction factor and u_b is the bulk velocity. Equation (3) is based on

- laminar flow theory for prediction of the molecular mass transfer rate.
- Reynolds analogy for prediction of the turbulent mass transfer rate in the core region.

- Higbie's penetration theory for prediction of the turbulent transfer rate in an intermediate region.
- Ruckenstein's penetration boundary layer flow model for prediction of the turbulent transfer rate in the wall region.

Equation (3) is rather impractical to use, and there exist other simpler correlations than Eq. (3), to predict the mass transfer coefficient with the same accuracy. For solutions at high Schmidt numbers, the shear stress can be considered to be constant in the concentration boundary layer. Then dimensional analysis gives

$$k = u^* \cdot f(Sc) \quad (4)$$

The most reliable mass transfer data at high Schmidt numbers are probably those of Shaw and Hanratty (1977), who experimentally obtained the following correlation.

$$k = 0.0889 \cdot u^* \cdot Sc^{-0.704} \quad 700 < Sc < 37000 \quad (5)$$

which is consistent with Eq. (4).

Equation (5) is probably valid also at $Sc < 37000$ but not at small Schmidt numbers, since then the shear stress varies within the concentration boundary layer, and causes the mass transfer coefficient also to be dependent on the size of the duct.

At $Sc < 300$ the correlations of Petukhov (1970), Eq. (6), and Notter and Sleicher (1972), Eq. (7), are recommended

$$k = \frac{\frac{f}{2} \cdot u_b}{1.07 + 12.7 \frac{\sqrt{f/2} \cdot (Sc^{\frac{2}{3}} - 1)}{2}} \quad 0.5 < Sc < 300 \quad (6)$$

$$\frac{k_d}{D} = 5 + 0.015 \cdot Re^a \cdot Sc^b \quad 0.5 < Sc < 300 \quad (7)$$

$$a = 0.88 - \frac{0.24}{4 + Sc}$$

$$b = \frac{1}{3} + 0.5 \exp(-0.6 \cdot Sc)$$

where Re is the Reynolds number.

At $300 < Sc < 700$, Eq. (5) and Eqs. (6) and (7) overpredict and underpredict, respectively, the mass transfer coefficient. In this range the correlations of Deissler (1961), Eq.(8a), and Ruckenstein (1967), Eq. (8b), are recommended.

$$k = 0.112 \cdot u^* \cdot Sc^{-\frac{3}{4}} \quad 300 < Sc < 700 \quad (8a)$$

$$k = 0.074 \cdot u^* \cdot Sc^{-\frac{2}{3}} \quad 300 < Sc < 700 \quad (8b)$$

However, in practical applications the velocity and concentration profiles are not always fully developed. Duct geometries other than circular ones are used and sometimes the membrane surface is fairly rough. In ultrafiltration the physical properties are often concentration dependent and in both reverse osmosis and ultrafiltration there is a transverse flow through the wall. In reverse osmosis and ultrafiltration the Schmidt numbers of the processed solutions range from approximately 500 to over 10^5 . In turbulent flow and at such high Schmidt numbers, the concentration boundary layer is so thin that within it the shear stress is practically constant. For these conditions the present work investigates how the mass transfer coefficient is dependent upon rough walls, the entrance region, turbulent Reynolds numbers below 10^4 , noncylindrical geometry, variable physical properties and permeable walls.

Rough walls

Rough walls have been considered, where the roughness elements do not contribute to any mass transfer resistance.

Dimensional analysis gives

$$k^+ = k(Sc, e^+, \text{roughness geometry}) \quad (9)$$

where k^+ is k/u^* and e^+ is the dimensionless height of the roughness elements ($e^+ = e \cdot u^* / \nu$). Equation (9) is confirmed by the experimental data of Dawson and Trass (1972). The higher the Schmidt number the lower e^+ for which k^+ is affected. At $e^+ \approx 10$, k^+ reaches its maximum value and at $e^+ > 10$ the correlation for k^+ is

$$k^+ = \text{const.} \cdot e^{+ -0.237} \cdot Sc^{-0.59} \quad e^+ > 10 \quad (10)$$

The constant in Eq. (10) is dependent on the roughness geometry and is in the order of 0.2. The exponent of e^+ may also be slightly dependent on the roughness geometry.

At $e^+ < 10$, k^+ increases with increasing e^+ because the higher the value of e^+ the more will whirls penetrate all the way to the wall. This is important with respect to the mass transfer rate for a solution with a low mass diffusivity. The momentum transfer is not so much affected by such low values of e^+ , because of the high momentum molecular diffusivity. At $e^+ \approx 10$, k^+ is increased by a factor of approximately 2.5 and 3.3 at $Sc = 400$ and 4600 respectively.

At $e^+ > 10$, the friction factor is also greatly affected by the roughness. The correlation for the dimensional mass transfer coefficient according to the data of Dawson and Trass is

$$\frac{k_d}{D} = 0.052 \left(\frac{e}{d_h}\right)^{-0.092} \cdot Re^{0.763} \cdot Sc^{0.41} \quad e^+ > 10 \quad (11)$$

Thus the mass transfer coefficient for rough walls and at high Schmidt numbers has about the same power dependence on Re and Sc as for smooth walls and low Schmidt numbers. This is because at rough walls, the whirls penetrate closer to the wall and reduce the effect of low mass diffusivity.

Entrance Region

Considered first are the conditions of a fully developed velocity profile at the commencement of the mass transfer section. For this case it is shown that at $x^+ < 500$, the correlation for the local dimensionless mass transfer coefficient is

$$k^+ = 0.54 \cdot x^{+ -\frac{1}{3}} \cdot Sc^{-\frac{2}{3}} \quad x^+ < 500 \quad (12)$$

are needed to determine the conditions at $600 < Sc < 2000$.

Noncircular Ducts

In noncircular ducts turbulence first occurs in the more open parts and, at higher Reynolds numbers, gradually extends to more closed parts of the duct. This causes the transition between laminar and turbulent flow to be not so sharp as for circular duct flow. The wall shear stress varies over the periphery, which causes secondary flows. These secondary flows are negligible in the concentration boundary layer at high Schmidt numbers, but the local mass transfer coefficient is proportional to the square root of the local shear stress and therefore also varies over the periphery. As an example, for square ducts the wall shear stress and the mass transfer coefficient at $Re \approx 3 \cdot 10^4$ reach their highest values at a point about half the distance from the centre of the wall to the corner. The value of the wall shear stress at this point is about 9% higher than its average peripheral value.

If the shear stress and the mass transfer coefficient are calculated from correlations valid for circular duct flow, the error in the resulting mass transfer coefficient will be,

- for square and rectangular ducts, less than 5% at $Sc > 500$ and $Re > 10^4$.
- for equilateral triangular ducts, less than 3% at $Sc > 500$ and $Re > 5 \cdot 10^4$.
- for concentric cylindrical annular ducts with the ratio $\frac{\text{inner radius}}{\text{outer radius}} > 0.5$, less than 7% at $Sc > 500$ and $Re > 10^4$.

Variable Physical Properties

In reverse osmosis and ultrafiltration there is no temperature variation in the cross section of the duct, and for turbulent flow the shear stress is practically constant in the concentration boundary layer. Hence in the concentration boundary layer, the physical properties can vary only with the solute concentration. Because the concentration boundary layer is very thin, the wall shear stress is not noticeably influenced by concentration dependent physical properties, and the bulk values should be used in calculating the shear stress.

It is not possible to use heat transfer data to obtain correlations for the mass transfer coefficient at variable physical properties. A theoretical analysis with unproved assumptions gives the result

$$k \approx 0.0889 \cdot u^* \cdot Sc_w^{-0.704} \quad Sc > 700 \quad (15)$$

where the subscript w denotes the quantity evaluated at the wall conditions. More experimental work is needed, however, to justify or disprove Eq. (15).

Permeable Walls

At permeable walls the mass transfer coefficient is defined according to Eq. (16)

$$j_w = k(c_b - c_w) + v_w \cdot c_w \quad (16)$$

where v_w denotes the permeate flux. Equation (16) also defines the mass transfer coefficient at nonpermeable walls, where v_w is zero. With this definition the mass transfer coefficient is always a measure of the concentration gradient near the wall

$$k = \frac{D \left(\frac{dc}{dy} \right)_w}{c_b - c_w} \quad (17)$$

The eddy diffusivity model is used to see how the permeate flow influences the mass transfer coefficient. It is realistic to assume that the eddy diffusivity near the wall is not influenced by the small permeate flux which occurs in reverse osmosis and ultrafiltration. Thus with constant physical properties a theoretical analysis gives the same result as the frequently used but unrealistic film theory

$$\frac{v_w}{k} = 1 - \exp\left(\frac{-v_w}{k(v_w=0)}\right) \quad (18)$$

where $k(v_w=0)$ is the mass transfer coefficient at the same conditions but with no permeate flux. Physically, Eq. (18) means that the concentration gradient at the wall increases with increasing permeate flux. Mizushima et. al. (1972) have experimentally confirmed Eq. (18) in the range $-0.9 < \frac{v_w}{u_*} \cdot 10^3 < 0.5$.

The connection between the permeate flux and solute concentrations is obtained from Eq. (19)

$$v_w = k(v_w=0) \cdot \ln \frac{c_w - c_p}{c_b - c_p} \quad (19)$$

where c_p is the solute concentration in the permeate. Equations (18) and (19) are valid at constant physical properties. The equations are not valid near the gelling concentration of the solute where the physical properties are strongly concentration dependent.

It is shown that for a typical reverse osmosis membrane without fouling, the permeate flux is mainly dependent on the membrane permeability, so making the mass transfer coefficient of only minor importance. If the permeate flux is increased by an increase in the static pressure difference, the solute rejection increases if $v_w < k(v_w=0)$ and decreases if $v_w > k(v_w=0)$.

For an ultrafiltration plant with specified permeate flux and wall solute concentration, the energy consumption per unit volume permeate is very slightly dependent on the hydraulic diameter of the channel. If the hydraulic diameter is doubled, the energy consumption increases by 10%. Of much greater importance is the pressure drop efficiency, i.e. that part of the total pressure drop in the recirculation loop which is over the active membrane area.

ACKNOWLEDGEMENTS

This work has been carried out at the Department of Chemical Engineering, Lund Institute of Technology. Papers I and III of this thesis have been financed by the Foundation for Membrane Technology, Sweden and the Swedish Board for Technical Development. Paper II of this thesis has been financed mainly by Alfa-Laval AB.

I wish to thank Dr. Alexander Klima for introducing me to the field of membrane technology, and my present and former colleagues for interesting discussions throughout the years of study.

I also thank H.-O. Jönsson for the skilful drawings and S. Dahlström for the excellent typing of my manuscripts.

NOTATION

c	solute concentration	kg/m ³
d _h	hydraulic diameter	m
D	mass diffusivity	m ² /s
e	height of roughness	m
e ⁺	e · u [*] /ν	
f	fannings friction factor, $\tau_w / \frac{1}{2} \rho u_b^2$	
j	solute flux in direction to the wall with respect to stationary axis	kg/m ² s
k	mass transfer coefficient	m/s
k ⁺	k/u [*]	
L	axial length	m
u _b	bulk velocity	m/s
u [*]	friction velocity $\sqrt{\tau_w / \rho} = u_b \cdot \sqrt{f/2}$	m/s
v	permeate flux	m/s
x	axial distance	m
x ⁺	x · u [*] /ν	
y	transverse distance from the wall	m
ε _q	eddy diffusivity of mass	m ² /s
ν	momentum diffusivity (kinematic viscosity)	m ² /s
ρ	density	kg/m ³
τ _w	wall shear stress	N/m ²

Sc	Schmidt number ($\frac{\nu}{D}$)
Re	Reynolds number ($\frac{d_h \cdot u_b}{\nu}$)

Subscripts

b	bulk value
p	permeate value
w	wall value
∞	asymptotic value

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Research report

MEMBRANE TECHNOLOGY RESEARCH AT LUND INSTITUTE OF TECHNOLOGY

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(Received June 18, 1978)

SUMMARY

Membrane technology research at Lund Institute of Technology, Sweden, is financed by a foundation consisting of 11 industries and the Swedish Board for Technical Development (STU) on a fifty-fifty basis. Several pilot plants are available, some of which are transportable for field tests.

Experimental investigations performed up till now are mainly in the following fields: dairy projects, dewatering of liquid foods, effluent treatment and water purification. Basic research is mainly devoted to concentration polarization studies.

Institutional research and development in the membrane field started in Sweden in 1970 at the Lund Institute of Technology. A membrane technology group of about six has been formed by the Divisions of Chemical Engineering and Food Engineering. The unit operations of reverse osmosis and ultrafiltration are studied from a chemical engineering viewpoint, and only little attention has been paid to membrane manufacture.

Already from the start the work has been devoted to both basic research and to practical, industrial problems, the latter resulting originally from a pronounced intention by the governmental Swedish Board for Technical Development to promote industrial applications of this new technology. Accordingly, in 1972 the Board financed the building of a "membrane station" consisting of flexible pilot plants for RO and UF. Fully automated, these rigs can be adjusted to work with most modules on the market. Later this equipment was extended to the range described below.

In order to direct this process development work in the interest of

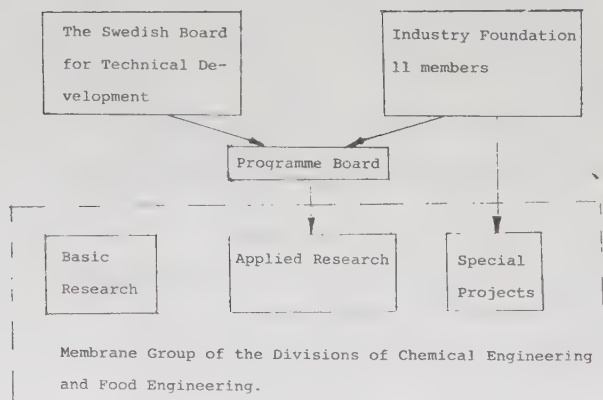


Fig. 1. Organizational scheme.

Swedish industry, the financing form has been changed recently. The Board mentioned above today shares the responsibility and the costs (on a fifty-fifty basis) with a foundation consisting of 11 industries. This organization has a Programme Board which plans the work. Even if this is directed by the member industries, the programme is general and aimed at process development work of interest to as many members as possible. A great deal of work is devoted to information service to the members.

EQUIPMENT

The equipment includes several pilot plants as well as laboratory cells for UF and RO. The pilot plants have all been designed and installed by people at our divisions. The first two, ready for use in spring 1973, were for UF (circulation flow rate 250 l/min, pressure 1 MPa) and for UF/RO (271/min, 8 MPa) (1). By now eight pilot plants are available, the largest with a maximum circulation flow rate of 1200 l/min (Table I). The variety of flow rates is due to the wish to use as many different types of membrane modules as possible. Some of the plants are easily transported for field tests (Fig. 2).

It is possible to arrange the plants both for continuous circulation and draining (feed and bleed) as well as for batch and single pass processes. The instrumentation is rather advanced. Data such as pressure, pressure drop, flow rates, temperature is recorded and controlled. Since the experiments are sometimes run continuously for 24 hours and more with a minimum of manual operation, some of the plants are provided with alarm switches for level, temperature and pressure.

We tried our best to get a sanitary design, avoiding threads and dead-ends, and choosing components made from stainless steel. There are some

TABLE I
PILOT PLANTS FOR UF AND RO

	<i>Circulation flow rate</i> <i>l/min</i>	<i>Max pressure</i> <i>MPa</i>
RO	0–15	6
RO	0–20	6
UF	0–10	0.7
UF	0–20	1
UF	0–80	0.18
UF	0–250	1
UF/RO	0–27	8
UF/RO	0–1200	6

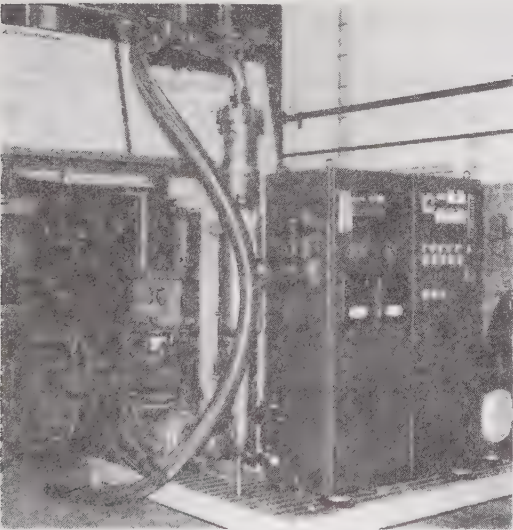


Fig. 2. Transportable UF/RO pilot plant for high flow velocities (circulation flow rate 0–1200 l/min).

problems in finding sanitary high pressure components since high pressure necessary in reverse osmosis is seldom used in the food industry.

APPLIED RESEARCH

The work is mainly concentrated on fields which had not been thoroughly investigated earlier. Accordingly no work is performed on for instance desalination of sea- or brackish water. On the other hand, for instance waste waters are mostly specific for each processing plant which necessitates special investigations.

The Swedish anti-pollution laws are gradually becoming more restrictive which forces industrial plants to reduce the waste discharge. In some cases RO or UF may be considered as suitable techniques. The membrane station staff may then be approached for advice and, if necessary, experimental investigation of the actual water is made.

In that way a number of experimental investigations have been ordered by different Swedish companies. In some cases investigations have been initiated by ourselves.

Experimental investigations performed up till now are mainly in the following fields:

Dairy projects

- separation of different valuable constituents from whey (2);
- study of economics and operating experience of existing plants using whey concentration by RO (3);
- UF-concentration of yoghurt, skim milk and low fat milk;
- RO-concentration of skim milk (4);
- study of the possibility of collecting and concentrating (RO) dairy rinse water (5);

Dewatering of liquid foods

Investigation of

- UF of blood plasma, hemoglobin solution and hemolyzed blood (6);
- UF of leaf protein juice, (7, 8) yeast autolyzate and wheat hydrolyzate;
- RO of apple juice.

Effluent treatment

Investigations of the suitability of using RO or UF for concentration of the contaminants in

- potato fruit water (9);
- leach water from the production of rapeseed protein;
- effluents from the fish industry (10);
- spent mash, beetroot condensate and cucumber liquor;
- cleaning solutions;
- effluents from bleaching plants in the pulp industry;
- effluents from textile dyehouses;
- effluents containing nitrotoluenes, polyoles, ammonium fluoride.

Water purification

- softening of tapwater;
- purification of poolwater.

Some of the projects are described below.

UF of effluents from pulp bleaching plants

In the pulp bleaching process the lignin in the pulp is first chlorinated

and then dissolved by an alkaline solution. Hitherto this alkaline solution containing chlorinated lignin compounds has been directly discharged to the recipient. Typical data for such an effluent appears in Table II.

TABLE II

TYPICAL DATA FOR AN EFFLUENT
FROM A PULP BLEACHING PLANT

Organics	4 kg/m ³
NaCl	2 kg/m ³
Colour	10 kg Pt/m ³
Temperature	60°C
pH	11

The polluting effects of this effluent is caused mainly by the high molecular fraction of the organics. This is strongly coloured and it is hard to biodegrade. For some years there has been pressure on the Swedish pulp industry from the National Swedish Environment Protection Board to reduce the effluents.

By commission of EKA Bohus—Sweden, a preliminary investigation of the suitability of fractionating the effluents by UF or RO was made. The organics should be concentrated while the sodium chloride should pass through the membrane. For the task four different membranes were tested: one CA-membrane rated 65% NaCl-rejection and three UF-membranes from Amicon with rated cut-off 500, 1,000 and 10,000 respectively. Some results are given in Table III.

TABLE III

DATA FROM FRACTIONATION OF UNCONCENTRATED EFFLUENT FROM A
PULP BLEACHING PLANT

Membrane	Flux l/m ² h	Rejection %		
		NaCl	Colour	COD
RO-CA pH 7, 30°C, 4 MPa	67	62	98.7	92
UF cut off 500, 50°C, 0.5 MPa	19	15	98.6	94
UF cut off 1,000, 50°C, 0.5 MPa	67	0	97.7	90
UF cut off 10,000, 50°C, 0.5 MPa	208	0	94.4	87

For all the membranes the colour- and COD-rejections increased with increasing degree of concentration, and the organics in the permeates were easily biodegraded.

EKA considered the results very promising and has later on developed

an UF-process with high flux membranes ($100\text{--}150\text{ l/m}^2\text{, h}$). If necessary the permeate is further treated biologically. The process has now reached a commercial stage. The UF-modules used are DDS modules type UF 35 with DDS-GR membranes. The permeate amounts to about 96% of the feed.

Dyehouse effluents

Effluents from dyehouses contain dyestuffs from concentrated spent dyeing baths as well as dilute rinsing water. All these effluents are presently discarded to the recipient, sometimes after biological treatment. Some years ago the Swedish textile industries nominated a working party to examine possibilities of decreasing pollution from dyehouses. By commission of this working party we have investigated possibilities to use RO or UF in this field.

Results from work on concentration of spent dyeing baths by RO is reported by Brandon and co-workers at Clemson University. Based on experiences from this investigation we decided to test representative effluents by means of UF-membranes. UF-membranes from Amicon with rated cut off $500\text{--}20,000$ were used. The following results were reached:

- Some of the effluents gave so low permeate fluxes that UF or RO concentration could not be regarded as reasonable.
- In some of the effluents the salt content was too high to render RO concentration possible.
- In all tests the colour rejection exceeded 90% and in some cases up to 98% was achieved.
- In most cases the total cost was lower for a spiral module RO plant than for an UF plant. This is due to the higher capital costs for an UF plant.

However, the total treatment cost in most cases exceeded $0.5\$/\text{m}^3$ permeate and this cost was not considered acceptable for this special application.

Effluents from polyalcohol production

A Swedish petrochemical factory has waste water containing mainly polyols and propylene glycol at low concentrations. The water, $7\text{ m}^3/\text{day}$, is now incinerated. Concentration by evaporation is not possible due to foaming. The cost for incineration is very high and is estimated at more than $40\$/\text{m}^3$. Preliminary tests on possibilities of concentrating the waste water by RO before incineration has recently been performed at our lab.

Two types of membranes were tested for this application, one CA membrane from DDS rated 99% NaCl-rejection and one PA-300 membrane from UOP.

Results from the tests are shown in Table IV. Permeate quality is measured in COD-content.

TABLE IV

DATE FROM WASTE WATER FROM POLYOL PRODUCTION $P = 6.0 \text{ MPa}$; $T = 25^\circ\text{C}$

Membrane	Initial flux $\text{l/m}^2, \text{h}$	COD-rejection
CA	14	97
PA-300	26	64

As appears PA-300 gave higher flux but lower COD-rejection than the CA-membrane. When the water had been concentrated four times with the CA-membrane the permeate flux had only decreased by 10%.

The permeate quality from the CA-membrane is such that it may be discharged which however is not the case for the permeate from the PA-300 membrane. According to the present very high price for the incineration, a RO-concentration plant would be very profitable for the company. However, further investigations in pilot scale are needed to demonstrate the possibilities.

Decolouration

When processing beetroots an intensively coloured waste water with a total solids content of about 7%, mainly sugar, is obtained. In Sweden the annual volume is about $2,500 \text{ m}^3$. The red colour is due to betanin (MW ca. 500), which is used as a food additive in some countries.

This type of waste water was successfully decolourized by using a CA-membrane with 85% NaCl rejection. Initial flux at $+30^\circ\text{C}$ and 5.5 MPa was approximately $40 \text{ l/m}^2, \text{h}$.

Fish waste waters

"Lutfisk" i.e. stockfish soaked in a solution containing calcium hydroxide and sodium carbonate, is a popular Scandinavian dish. During the soaking more than 50% of the fish proteins are lost into the process water. In Sweden the annual volume of such process waste water is about $18,000 \text{ m}^3$. Total solids content is about 0.5–1.5% and pH 11–12. The proteins are wasted today but experiments of collecting and utilizing the proteins which seem very promising are going on.

By precipitating the proteins with hydrochloric acid at the isoelectric point, about 70% of the proteins are recovered. It is, however, difficult to handle the large volumes of process waste water. Another problem is the large quantities of hydrochloric acid necessary for pH-adjustment.

UF seems to be an excellent method of processing this type of waste water and a tenfold volume concentration ratio is easily obtained. When

treating fivefold concentrated waters with hydrochloric acid, the protein recovery increases to 85%.

Typical average fluxes at 20°C and a tenfold concentration ratio are about 15 l/m², h (hollow fiber module, average pressure 0.12 MPa, membrane cut off 10,000) and 35 l/m², h (plate and frame module, average pressure 0.8 MPa, membrane cut off 22,000) respectively.

Rinse waters in food industries

Diluted process waters (rinse waters) are obtained when flushing the equipment with water before changing product or before cleaning. The rinse waters are often discharged today. Due to discharge restrictions and increasing costs for sewage treatment it might be attractive to collect, concentrate and reuse the rinse water.

As an example, we studied the dairy rinse water in two pipe systems in a dry milk factory. About 150,000 l skim milk/year or 5,000 kg protein, discharged at the moment, could be collected. A fraction with 4.5% total solids on an average was concentrated to 18% by reverse osmosis. Experiments at 3 MPa and 20°C gave initial permeate fluxes of 27 l/m², h at the best and final fluxes of 5 l/m², h. The permeate contained organic material corresponding to about 95 mg KMnO₄/l at pH 6.8 and 145 mg KMnO₄/l at pH 6.4.

The experiments showed that about \$5,000/year could be saved if the rinse water is concentrated by reverse osmosis, evaporated, dried and sold as fodder instead of discharged.

BASIC RESEARCH

Applied research as described above has to be based on and combined with basic research. During the years, several research projects have been integrated in the work of the group. This research is mainly effected by students (master of science thesis) and post-graduates. Two researchers are working on their doctoral theses.

The main theme of the research is concentration polarization. In one thesis a hydraulic flow model has been developed for ultrafiltration of macromolecules (11). Different ways of diminishing the influence of the polarization layer also have been studied (12). With the same aim, studies have been made of flow conditions in a recently developed rotating module (13). Further research has been done regarding scaling up, plant layout and optimization (14), energy studies (3, 4, 15) cleaning, etc.

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Omvänd osmos för koncentration av vassle-inventering och drifts- erfarenheter

Concentration of whey by reverse osmosis-inventory and operating experiences

Umkehrosmose für Molkekonzen- trierung-Inventur und Betriebserfahrungen

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Omvänd osmos för koncentration av vassle – inventering och driftserfarenheter

Concentration of whey by reverse osmosis – inventory and operating experiences

Umkehrosmose für Molkekonzentrierung – In- ventur und Betriebserfahrungen

Under 1970-talet har ett flertal artiklar publicerats som behandlar användning av omvänd osmos (RO) inom mejeriindustrin. Dessa behandlar både teoretiska och praktiska synpunkter och dessutom ekonomin. Vad som saknas är uppgifter om driftserfarenheter vid RO-anläggningar från andra än tillverkaren av anläggningen.

Denna artikel behandlar en inventering av levererade RO-anläggningar för vasslekoncentrering samt uppgifter om driftserfarenheter från några av dessa. Uppgifter om driftserfarenheterna har erhållits vid besök på aktuella mejerier. Vidare behandlas kostnaderna vid vasslekoncentrering med RO.

Inventering

Följande företag tillfrågades under oktober 1976 om de levererat RO-anläggningar för kommersiellt bruk:

Abcor
Büttner-Schilde-Haas
De Danske Sukkerfabrikker (DDS)
Osmonics
Paterson Candy International (PCI)
Rhône Poulenc
Union Carbide
Universal Oil Products (UOP)
Wafilin

Av dessa hade DDS, PCI, UOP och Abcor levererat RO-anläggningar till mejeriindustrin.

Abcor har levererat 5 st RO-anläggningar med sammanlagd membranarea av 150 m² till livsmedelsindustrin i Japan. Någon eller några av dessa kan vara inom mejeriindustrin. Ytterligare har Abcor levererat en RO-anläggning i USA som koncentrerar 120 m³/d laktos (förmodligen UF-permeat).

UOP har levererat 2 st RO-anläggningar för koncentration av vassle, en till Nya Zeeland 1968* och en till USA 1974.

Tabell 1 visar RO-anläggningar för koncentration av vassle som DDS och PCI har levererat eller kommer att leverera i början av 1977.

Samtliga dessa anläggningar är placerade i Västeuropa. DDS har

During the 1970's a lot of articles have been published which deal with the utilisation of Reverse Osmosis (RO) for dairy products. These articles treat both theoretical and practical aspects as well as economy. However, operating experiences from RO-plants described by others than plant manufacturers are missing.

This article concerns a number of delivered RO-plants for whey concentration and operating experiences from some of these.

In addition, the costs of whey concentration by RO are treated.

Inventory

The following companies were asked in October 1976 if they had delivered any commercial RO-plants:

Abcor
Büttner-Schilde-Haas
De Danske Sukkerfabrikker (DDS)
Osmonics
Paterson Candy International (PCI)
Rhône Poulenc
Union Carbide
Universal Oil Products (UOP)
Wafilin.

Of these DDS, PCI, UOP and Abcor had delivered RO-plants to the dairy industry.

Abcor has delivered 5 RO-plants with a total membrane area of 150 m² to the food industry in Japan. Some or more of these may be for the dairy industry. Abcor has also delivered a RO-plant to the USA which concentrates 120 m³ lactose per day (probably UF-permeate). UOP has delivered 2 RO-plants for whey concentration, one to New Zealand in 1968* and one to USA in 1974.

Table 1 shows RO-plants for whey concentration which DDS and PCI have delivered or will deliver in 1977.

Table 1: RO-plants delivered by DDS and PCI for whey concentration, which are ready or will be in operation in 1977.

Manufacturer	Number of plants	Total membrane area (m ²)
DDS	8	2000
PCI	5	1100

Im Laufe der 70-iger Jahre wurden über ein Dutzend Artikel publiziert, die die Anwendung der Umkehrosmose in der Molkereiindustrie zum Gegenstand haben. Sie behandeln theoretische und praktische Gesichtspunkte und auch ökonomische Fragen. Es fehlen jedoch Angaben über Betriebserfahrungen sozus. von »neutraler Hand« d.h. von Nichtherstellern.

Dieser Artikel gibt eine Übersicht über im Betrieb stehende RO-Anlagen für Molkekonzentrierung und berichtet über Betriebserfahrungen, die in einigen Fällen vom Verfasser persönlich gesammelt wurden. Ferner werden die Kosten der Molkekonzentrierung mittels RO behandelt.

Inventur

Die folgenden Unternehmungen wurden im Oktober 1976 hinsichtlich gelieferten kommerziellen RO-Anlagen befragt:

Abcor
Büttner-Schilde-Haas
De Danske Sukkerfabrikker (DDS)
Paterson Candy International (PCI)
Osmonics
Rhône Poulenc
Union Carbide
Universal Oil Products
Wafilin.

Von diesen haben DDS, PCI, UOP und Abcor RO-Anlagen an die Molkereiindustrie geliefert.

Abcor lieferte 5 RO-Anlagen mit insgesamt 150 m² Membranfläche an die Lebensmittelindustrie in Japan. Eine oder mehrere könnten an die Molkereiindustrie gegangen sein. Ferner lieferte Abcor eine RO-Anlage in die USA für die Konzentrierung von 120 m³/T Laktose (vermutlich UF-Permeat).

UOP lieferte 2 RO-Anlagen für Molkekonzentrierung, eine nach New Zealand im Jahre 1968*) und eine in die USA im Jahre 1974.

Tabelle 1 zeigt RO-Anlagen für Molkekonzentrierung, die von DDS oder PCI geliefert wurden oder im Laufe von 1977 geliefert werden.

* Die Anlage wurde von Havens geliefert, später wurde Havens von UOP aufgekauft.

* RO-anläggningen såldes av Havens 1968. Senare har UOP köpt Havens.

* The RO-plant was sold by Havens in 1968. Later UOP bought Havens.

Tabell 1. Av DDS och PCI levererade RO-anläggningar för koncentrer-
ring av vassle som är färdiga eller
kommer att tagas i drift under 1977.

Tillverkare	Antal anlägg- ningar	Summa membranyta (m²)
DDS	8	2000
PCI	5	1100

även levererat en RO-anläggning för
koncentrer-
ing av skummjolk och en
för koncentrer-
ing av UF-permeat.

Ovanstående visar att det är DDS
och PCI som fn är ledande vad gäller
anläggningar för omvänd osmos inom
mejeriindustrin. Första kommersiella
RO-anläggningen för vasslekoncentrer-
ing från DDS resp. PCI togs i drift
1972 resp. 1975. I fortsättningen
kommer endast anläggningar från
DDS och PCI att behandlas.

Systemutförande

RO-anläggningarna från PCI är
kontinuerliga, enkel passage enl
fig. 1.

RO-anläggningarna från DDS är
antingen kontinuerliga, kaskadkop-
plade enl fig. 2 eller satsvisa enl fig. 3.

I en typisk PCI-anläggning är
vasslens genomsnittliga uppehållstid
omkring sex minuter och uppehålls-
tidfördelningen är snäv, vilket gör
att temperaturen kan hållas vid 30°C
utan att bakterietillväxten blir för
stor. I DDS-anläggningarna är uppe-
hållstiderna längre p.g.a. recirkulation-
en varför temperaturen där är om-
kring 10°C för att hålla nere bak-
terietillväxten. Nackdelen med lägre
temperatur är dels kostnaderna för
kylning dels att permeatflödet min-
skar med temperaturen. Både DDS

Fig. 1. *Principschema
över kontinuerlig,
enkel passage
RO-anläggning
från PCI.*

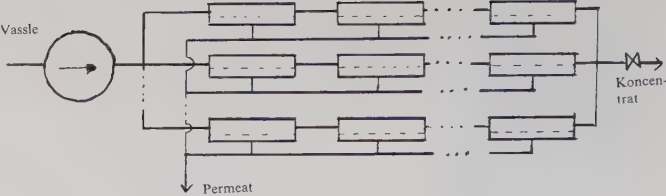


Fig. 1. *System design for a continuous single passage RO-
plant from PCI.*

Abb. 1. *Prinzipschema einer kontinuierlichen RO-Anlage
von PCI mit einfacher Passage.*

All these plants are situated in We-
stern Europe. DDS also has delivered
one RO-plant for skim milk concen-
tration and one for UF-permeate concen-
tration.

The above mentioned shows that,
at present, DDS and PCI are leading
in the field of RO for the dairy in-
dustry. The first commercial RO-
plants for whey concentration from
DDS and PCI started in 1972 and
1975 respectively. In the following
text plants from DDS and PCI alone
will be discussed.

System performance

PCI delivers continuous single pas-
sage plants according to fig. 1.

DDS delivers either continuous re-
circulating plants according to fig. 2
or batch-wise plants according to
fig. 3.

For a typical PCI-plant the average
residence time of whey is about six
minutes and the residence time di-
stribution is close. Because of the low
residence time the temperature can
be 30°C without too great bacterial
growth. In the DDS-plants the resi-
dence time is longer because of the
recirculation. The maximum tempe-
rature is then about 10°C to keep
down bacterial growth. The drawback
of low temperature is, on one hand

Tabelle 1. Von DDS oder PCI gelie-
ferte RO-Anlagen für die Molkekon-
zentrierung, die in Betrieb sind oder
im Laufe von 1977 in Betrieb gesetzt
werden.

Hersteller	Anzahl der Anlagen	Totale Membran- fläche
DDS	8	2000
PCI	5	1100

Alle diese Anlagen befinden sich
in Westeuropa. DDS hat auch eine
RO-Anlage für die Konzentrierung
von Magermilch und eine für Kon-
zentrierung von UF-Permeat gelie-
fert.

Aus dem bisher gesagten geht her-
vor, dass z.Z. DDS und PCI auf dem
Gebiet der Umkehrosmose in der
Molkereiindustrie führend sind. Die
ersten kommerziellen RO-Anlagen
für Molkekonzentrierung wurden von
DDS bzw. PCI i.d. Jahren 1972
bzw. 1975 in Betrieb genommen. Im
folgenden werden nur noch Anlagen
von DDS und PCI behandelt werden.

Apparatgestaltung

Die RO-Anlagen von PCI sind kon-
tinuierlich, mit einfacher Passage lt.
Abb. 1.

Die RO-Anlagen von DDS sind
entweder kontinuierlich, mit Kaska-
denschaltung laut Abb. 2 oder sats-
weise arbeitend lt. Abb. 3.

In einer typischen PCI-Anlage ist
die durchschnittliche Verweilszeit un-
gefähr 6 Minuten und das Verweils-
zeitspektrum ist schmal – aufgrund
dessen kann die Temperatur bei 30°C
gehalten werden, ohne dass das Bak-
terienwachstum zu gross wird. In den
DDS-Anlagen ist infolge der Rezir-
kulation die Aufenthaltszeit länger,
weshalb die Temperatur bei ca. 10°C
gehalten werden muss, um das Bak-
terienwachstum niedrig zu halten. Die
niedrigere Temperatur wirkt nach-
teilig, teils wegen der Kosten für die
Kühlung und teils wegen der Vermin-
derung des Permeatflusses infolge
Temperatursenkung. Beide, DDS und
PCI dimensionieren jedoch ihre An-
lagen aufgrund eines Permeatflusses
von 20–25 l/m² h für ein Molkekon-
zentrieren im Verhältnis von 1:2.

Man kann auch eine kontinuier-

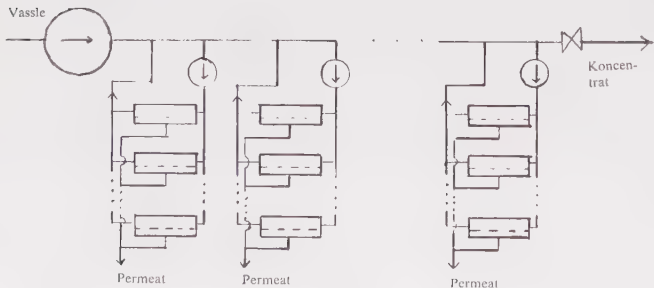


Fig. 2. *Principschema
över kontinuerlig
kaskadkopplad
RO-anläggning
från DDS.*
Fig. 2. *System
design for a
continuous
recirculating
RO-plant from DDS.*
Abb. 2. *Prinzipschema
einer kontinuierlichen
DDS-Anlage mit
Kaskadenkopplung.*

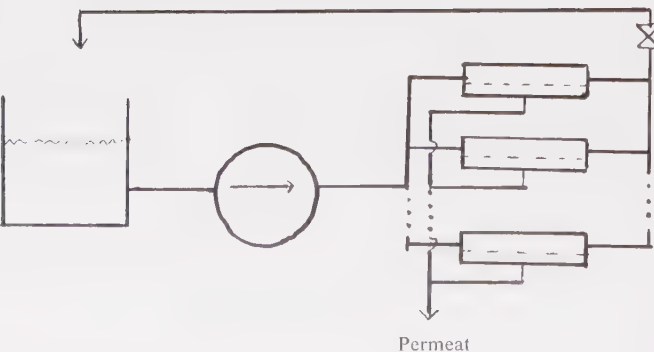


Fig. 3. *Principschema
över satsvis
RO-anläggning
från DDS.*
Fig. 3. *System
design for a
batch-wise
RO-plant from DDS.*
Abb. 3. *Prinzipschema
einer diskontinuierlichen
DDS-Anlage.*

och PCI dimensionerar dock sina anläggningar efter ett permeatflux av 20–25 l/m² tim vid koncentration av vassle i förhållandet 1:2.

Driftserfarenheter

Även en kontinuerlig, enkel passage anläggning går att använda för satsvis drift. PCI har levererat en RO-anläggning som koncentrerar vassle kontinuerligt varvid koncentratet kyles och lagras i en tank. När vasslen är slut för dagen koncentreras vasslekoncentratet satsvis i samma anläggning under natten. Detta sker givetvis vid låg temperatur.

När vasslens torrsubstanshalt ökar så ökar vasslens viskositet och minskar de olika lästa ämnens diffusivitet. P.g.a. detta minskar permeatflux med ökad torrsubstanshalt. Som överslagsberäkning kan användas

$$\text{Permeatflux} = \frac{\text{konstant}}{\text{torrsubstanshalt}}$$

Detta förorsakar att kostnaden per avlägsnad mängd permeat ökar med koncentreringsgraden.

Det är knappast ekonomiskt att med RO koncentrera vassle till över 25% torrsubstanshalt. Vanligen dimensioneras RO-anläggningarna för en koncentreringsgrad mellan 1:2 och 1:4.

Besök gjordes vid fem st RO-anläggningar, tre från DDS (satsvisa) och två från PCI.

Driftstiden för RO-anläggningarna var vanligen 5–10 tim/dygn 4–6 dagar i veckan.

En av PCI-anläggningarna var endast drygt en månad gammal och under denna tid hade allt fungerat bekymmersfritt. De övriga RO-anläggningarna var mellan 12 och 20 månader gamla. I alla anläggningarna används en kolvump med tre eller fem kolvar som matarpump. I början hade två DDS-anläggningar problem med vibrationer och tryckpulsationer i ledningarna, vilket gjorde att svetsskarvar sprack. Detta avhjälpes med att placera en invändigt teflonbelagd gummislang mellan pump och rörledning, och i ett fall dessutom en tryckackumulator. En PCI-anläggning hade i början problem med bakterietillväxt. Detta berodde på två saker, dels att personalen behandlade vasslen som en avfallsprodukt i st f som en råvara dels att den använda pumpen inte var hygieniskt utformad. Efter modifieringar av pumpen och instruktioner till personalen lär inga bakterieproblem förekomma längre. Ovan beskrivna problem kan betraktas som barnsjukdomar och är snart avhjälpda.

Koncentreringsgraden styrdes i de satsvisa DDS-anläggningarna huvudsakligen genom driftstiden. När önskad koncentreringsgrad erhållits av-

the cooling cost, on the other that the permeate flow decreases with temperature. However both DDS and PCI design their RO-plants for a permeate flux of 20–25 l/m² h for whey concentration in ratio 1:2.

It is possible to use a continuous single passage plant for batch-wise operation. PCI has delivered a continuous single passage plant where the concentrate is cooled and goes to a storage tank. When there is no raw whey left for the day, the whey concentrate is further concentrated batch-wise in the same plant during the night. The temperature is then of course low.

As the dry solids content of the whey increases, the viscosity of the whey increases and the diffusivities of the solutes decrease. As a result, the permeate flux decreases as the dry matter content increases, so the total cost per m³ permeate will increase with the concentration ratio.

Roughly:

$$\text{Permeate flux} = \frac{\text{constant}}{\text{dry matter content}}$$

It is hardly economical to concentrate whey by RO to more than 25% dry matter content. Usually, RO-plants for whey concentration are designed for a concentration ratio between 1:2 and 1:4.

Operating experiences

Five RO-plants were visited, three DDS-plants (batch-wise) and two PCI-plants.

The operating time of the RO-plants were usually 5–10 hours/day, 4–6 days/week.

One of the PCI-plants had been in operation for little more than one month, and during this time no trouble had occurred. The other RO-plants were between 12 and 20 months old. In all plants the feed pump was a plunger pump with three or five plungers. In the beginning two DDS-plants had problems with vibration and pressure pulsation in the tubes, which caused joint bursting. These problems were solved by putting an inside PTFE-coated flexible hose and in one case also a pressure accumulator between the pump and the tube. At start one PCI-plant had problems with bacterial growth. These were partly caused by the personnel treating the whey as waste instead of a raw product, partly by the feed pump being insanitary. After instructions to the personnel and modification of the pump there were no problems any more.

For the batch-wise DDS-plants the concentration ratio was controlled mainly by the operating time. When the desired concentration ratio is reached the feed pump is shut down. For the continuous PCI-plants the

liche Anlage mit einfacher Passage diskontinuierlich fahren. PCI hat eine RO-Anlage geliefert, die Molke kontinuierlich konzentriert, wobei das Konzentrat gekühlt und gelagert wird. Nach Verbrauch der Tagesration wird das Konzentrat in der selben Anlage während der Nacht diskontinuierlich aufkonzentriert. Dies geschieht natürlich bei niedriger Temperatur.

Wenn der Trockensubstanzgehalt der Molke steigt so steigt auch die Viskosität der Molke und die Diffusivität der div. gelösten Stoffe verringert sich. Aufgrund dessen verringert sich der Permeatfluss mit steigendem Trockensubstanzgehalt. Für eine Überslagsberechnung kann die folgende Formel angewendet werden:

$$\text{Permeatfluss} = \frac{\text{Konstante}}{\text{Trockensubstanz}}$$

Infolge dessen steigen die Betriebskosten pro entfernte Permeatmenge mit dem Konzentrationsgrad.

Es ist kaum ökonomisch, mit RO Molke über 25% Trockensubstanz zu konzentrieren. RO-Anlagen werden gewöhnlich für einen Konzentrierungsgrad von 1:2 bis 1:4 dimensioniert.

Betriebserfahrungen

Der Verfasser besuchte 5 RO-Anlagen, drei von DDS (satzweiser Betrieb) und zwei von PCI.

Die Betriebszeit für RO-Anlagen war gewöhnlich 5–10 h/T, 4–6 Tage/Woche.

Eine der PCI-Anlagen war nur einen Monat alt und während dieser Zeit funktionierte alles tadellos. Die anderen RO-Anlagen waren 12 bis 20 Monate im Gebrauch. In allen wird je eine Kolbenpumpe mit 3 oder 5 Kolben als Speisepumpe angewendet. Am Anfang gab es in zwei DDS-Anlagen Probleme mit Vibrationen und Druckschwankungen in den Rohrleitungen, wodurch einige Schweißnähte sprangen. Dem wurde abgeholfen durch das Anbringen eines mit Teflon gefütterten Gummischlauches bzw. in einem Falle zusätzlich eines Druckschlauches zwischen der Pumpe und der Rohrleitung. Bei einer PCI-Anlage gab es am Anfang Probleme mit Bakterienwachstum. Dies wurde durch zwei Momente hervorgerufen, teils behandelte das Personal die Molke nicht als Rohware sondern als Abfall und teils waren die Pumpen nicht hygienisch konstruiert. Nach dem Modifizieren der Pumpe und Instruktion des Bedienungspersonals sollen keine Bakterienprobleme mehr vorgekommen sein. Diese Probleme können als leicht eliminierbare Kinderkrankheiten betrachtet werden.

Der Konzentrierungsgrad wird bei

slutades koncentreringsgraden. I de kontinuerliga PCI-anläggningarna bestäms koncentreringsgraden huvudsakligen av flödet från matarpumpen och antalet moduler. Detta bestäms vid dimensioneringen av RO-anläggningen och ändras inte gärna sedan. Små justeringar av koncentreringsgraden går dock att utföra genom att variera trycket med koncentratventilen.

RO-anläggningarna kräver arbetskraft endast vid start och avstängning samt vid rengöring. Under drift räcker det med att operatörerna tittar till dem ibland.

Den tid operatören måste vara på plats och sköta anläggningen är per dygn ca. 1 tim för en DDS-anläggning och 1.5 tim för en PCI-anläggning. Orsaken till att PCI-anläggningen kräver mer tid är dels att operatören måste vara närvarande när tillflödet upphör och dels att det tar längre tid att förtränga vätskorna ur anläggningen.

Dygnscykeln är:

1. RO-anläggningen startar med vasslekoncentreringsgrad.
2. Koncentreringsgrad avslutas och kvarvarande vasslekoncentrat i anläggningen förträngs med vatten.
3. Rengöring.

Rengöringen utförs vid lämplig tid ur arbetskraftssynpunkt. Om vasslekoncentreringsgraden inte är färdig förrän vid arbetstidens slut kan den göras nästa morgon. Det är viktigt att rengöringen sker på rätt sätt. I annat fall minskar membranens livstid och kapacitet och vasslekoncentratets bakteriehalt ökar. I en DDS-anläggning som var 12 månader gammal skall samtliga membran bytas p.g.a. bl.a. lågt permeatflux. Detta beror förmodligen på att rengöringsrutinen varit felaktig de senaste månaderna.

Ingen av de anläggningar som besökts hade fått något membran utbytt. Både DDS och PCI garanterar ett års membranlivslängd vid vasslekoncentreringsgrad. Enligt uppgift från DDS är den genomsnittliga membranlivslängden 1.5 år för DDS-anläggningar.

Som ovan nämnts dimensionerar både DDS och PCI sina RO-anläggningar för ett genomsnittligt permeatflux av 20–25 l/m² tim vid koncentreringsgrad av vassle i förhållandet 1:2. Permeatflux minskar dock när membranens ålder ökar. Hur stor minskningen är beror till avsevärd del på skötseln av RO-anläggningen. Tabell 2 visar värden som uppmätts vid besöken på resp. RO-anläggning.

Data i tabell 2 är mestadels baserade på mätningar under en eller två dagar. Enligt personalen som skötte RO-anläggningarna var det inga

concentration ratio mainly depends on the feed flow rate and the number of modules. These are determined at the design of the plant. However it is possible to make small adjustments to the concentration ratio by controlling the pressure by means of the concentrate valve.

The RO-plants require labour only at start up, shut down and during cleaning. During operation it is enough for the operators to check the plants occasionally. The time required for a DDS-plant is about one hour a day and for a PCI-plant about one and a half hours a day. A PCI-plant requires more working hours because on one hand an operator has to be present when the feed terminates, on the other it takes a longer time to push the different liquids out of the membrane tubes.

The daily cycle is

1. The RO-plant starts up with whey concentration.
2. The concentration of whey ends, and the whey in the tubes is ejected using water.
3. Cleaning.

Cleaning is done when the labour supply is suitable. If the whey concentration is ready at the end of working hours, the cleaning can be done the next morning. It is important that the cleaning is correctly performed. If this is not so, the membrane lifetime and capacity will decrease and the bacterial content of the whey concentration increase. In a 12 months old DDS-plant all membranes will be changed because of, among other things, a low permeate flux. This is probably due to the cleaning routine being performed incorrectly during the last few months.

None of the visited RO-plants had changed any membrane. Both DDS and PCI guarantee a one year membrane life for whey concentration. According to information from DDS, the average membrane-life is 18 months for DDS-plants for whey concentration.

As mentioned above both DDS and PCI design their RO-plants for an average permeate flux of 20–25 l/m²h for a whey concentration ratio of 1:2. However, the permeate flux decreases with the increasing age of the membrane. Therefore the care and maintenance of the RO-plant will mean a lot. Table 2 shows data which were valid at the visits to the five RO-plants.

The data in table 2 are mainly obtained from one or two days measurements. According to the RO-plant operators there were no noticeable variations of the permeate flux

den diskontinuerlichen DDS-Anlagen hauptsächlich durch die Betriebsdauer gesteuert. Ist der gewünschte Konzentrierungsgrad erreicht, wird der Prozess beendet. In den kontinuierlichen PCI-Anlagen wird der Konzentrierungsgrad durch die eingespeiste Menge und Anzahl der Module bestimmt. Dies wird beim Dimensionieren der RO-Anlage festgelegt – Änderungen sind hier unerwünscht. Geringfügige Justierungen des Konzentrierungsgrades können jedoch durch Variieren des Druckes mit dem Konzentratventil durchgeführt werden.

RO-Anlagen erfordern Bedienung nur beim Start, Abstellen und beim Reinigen. Während des Betriebes genügt eine zeitweise Inspektion.

Die notwendige Bedienungszeit ist für eine DDS-Anlage 1 St. und 1,5 Stunden für eine PCI-Anlage. Die längere Bedienungszeit für die PCI-Anlage ist teils dadurch bedingt, dass der Operateur beim Aufhören des Zulaufes anwesend sein muss, und teils dadurch, dass es längere Zeit dauert, die Flüssigkeit aus der Anlage zu verdrängen.

Der Tageszyklus ist wie folgt:

1. Die RO-Anlage startet mit der Molkekonzentrierung.
2. Das Konzentrieren wird beendet und das im Apparat verbliebene Molkekoncentrat wird mit Wasser verdrängt.
3. Reinigung der Apparatur.

Das Reinigen der Apparatur muss nicht unmittelbar nach Arbeitsschluss erfolgen, soll aber fachmännisch richtig durchgeführt werden. Sonst verringert sich die Lebenslänge sowie die Kapazität der Membranen und der Bakteriengehalt des Konzentrates steigt. In einer DDS-Anlage, die 12 Monate in Betrieb war, müssen u.A. aufgrund des geringen Permeatflusses sämtliche Membranen ausgetauscht werden. Vermutlich ist dies durch eine unzureichende Reinigungsmethode in den letzten Monaten verursacht worden.

In keinen der besuchten Werke wurden Membranen ausgetauscht. Beide, DDS und PCI, garantieren eine einjährige Lebensdauer der Membranen für Molkekonzentrierung. Nach Angaben von DDS ist die durchschnittliche Lebensdauer der Membranen in ihren Anlagen 1,5 Jahre.

Wie oben erwähnt, dimensionieren beide, DDS und PCI, ihre Anlagen für eine Molkekonzentrierung im Verhältnis 1:2 für einen durchschnittlichen Permeatfluss von 20–25 l/m² h. Der Permeatfluss verringert sich jedoch mit zunehmendem Alter der Membran. Die Fluxverminderung beruht in beträchtlichen Masse an der

Tabell 2. Data från besökta RO-anläggningar. I samtliga fall är ingångstryck 40 bar.

Table 2. Data from visited RO-plants. In all cases inlet pressure is 40 bar.

Tabelle 2. Angaben von besuchten RO-Anlagen. In allen ist der Eintrittsdruck 40 bar.

Anläggning	Ålder (månader)	Permeatmängd (% av vasslevolym)	Genomsnittligt permeatflux (1/m ² tim)
DDS 1	20	40–50	17
DDS 2	16	44	16
DDS 3	12	50	13
PCI 1	1	43	22
PCI 2	15	30*	21*

Plant	Age (months)	Permeate volume (% of whey volume)	Average permeate flux (1/m ² h)

* Värdet hänför sig från mätning vid endast en tidpunkt.

* Value taken from just one measurement.

* Werte stammen von einer einzigen Messung.

märkbara förändringar av permeatflödet från dag till dag, så ovanstående data är troligen representativa för resp. anläggning vid denna tidpunkt. Anläggningen benämnd PCI 2 utgör ett undantag eftersom mätningar här utförts endast en gång och då var vassletemperaturen omkring 25°C. Normalt är vassletemperaturen 32°C i denna anläggning vilket ökar permeatflux med uppskattningsvis 20% jämfört med permeatflux vid 25°C. Mätningen utfördes i slutskedet av en dags drift. Anläggningen benämnd DDS 3 ger lågt permeatflux förmodligen beroende på att anläggningen rengjorts felaktigt de senaste månaderna. Vid jämförelse mellan RO-anläggningarna från DDS och PCI måste hänsyn tagas till att koncentrationegraden är högre för DDS-anläggningarna. T. ex. för anläggningen benämnd DDS 2 var genomsnittliga permeatflux omkring 20 l/m² tim då 30% av ursprunglig vasslevolym avlägsnats som permeat.

Ekonomi

Anledningen till att mejerierna skaffat sig RO-anläggningar var

- vasslen indunstades och kokades till brunost. Av olika skäl räckte inte indunstningskapaciteten till. Valet stod mellan att utöka antalet indunstningseffekter eller att förkoncentrera vasslen med omvänd osmos. Alternativet med omvänd osmos ansågs fördelaktigare. (Tre fall).
- Vasslen fick inte längre släppas ut i avloppet. Den koncentreras nu med omvänd osmos till 10% torrhalt och returneras som djurfoder tillsammans med skummjolk (1 fall).
- Vasslen transporteras till en central indunstningsanläggning. Ge-

from one day to another, so the data probably are typical for each RO-plant at that time. The plant named PCI 2 makes an exception as measurements here were only made once when the whey temperature was 25°C. Normally the whey temperature is 32°C in this RO-plant, which increases the permeate flux roughly 20%. The measurement was made at the end of the operation. The plant named DDS 3 has a low permeate flux, probably due to an incorrectly performed cleaning routine during the last few months. If the DDS- and PCI-plants are compared it has to be remembered that the concentration ratio is higher for the DDS-plants. For instance the plant named DDS 2 had an average permeate flux about 20 l/m²h when 30% of the raw whey volume had been removed as permeate.

Economy

The reasons for the dairies buying RO-plants were

- the whey was evaporated and then cooked to brown cheese. For different reasons the evaporator capacity was not enough. The choice was either to increase the number of evaporators or to preconcentrate the whey by RO. RO was regarded as being most profitable. (Three cases).
- It is no longer allowed to discard whey, so the whey is now concentrated to a dry matter content of 10% and then returned together with skim milk as cattle food. (One case).
- The whey is transported to a central evaporating plant. By preconcentrating by RO the costs for cooling, transport to and steam consumption at the central evapo-

Bedienung. In der Tabelle 2 sind Werte angeführt, die beim Besuch der einzelnen RO-Anlagen festgestellt wurden.

Die Daten in der Tab. 2 gründen sich meistens auf Messungen während 1 oder 2 Tagen. Nach Aussagen des Bedienungspersonals gab es keine von einem Tag auf den anderen merkbare Schwankungen im Permeatfluss, so dass die angeführten Daten als repräsentativ für den Zeitpunkt des Besuches angesehen werden können. Die mit PCI 2 bezeichnete Anlage bildet eine Ausnahme, da nur eine Messung durchgeführt wurde, und hier war die Molketemperatur 25°C. Normalerweise ist in dieser Anlage die Molketemperatur 32°C, wodurch der Permeatfluss schätzungsweise um 20% grösser wird als bei 25°C. Die Messung wurde am Ende der Tagsschicht durchgeführt.

Die als DDS 3 bezeichnete Anlage ergab geringen Permeatfluss, wahrscheinlich als Folge einer unsachgemässen Reinigung während der letzten Monate. Beim Vergleich mit den von DDS und PCI gelieferten Anlagen muss der in der DDS-Anlage erreichte höhere Konzentrationsgrad berücksichtigt werden. So war z.B. der durchschnittliche Permeatfluss für die mit DDS 2 benannte Anlage ungefähr 20 l/m² h wenn 30% des ursprünglichen Molkevolums als Permeat abgezogen wurde.

Ökonomie

Die folgenden Momente haben Molkereien veranlasst, RO-Anlagen anzuschaffen:

- die Molke wurde eingedampft und sogen. Braunkäse hergestellt. Aus verschiedenen Gründen reichte die Eindampfanlage nicht aus. Es bestand die Alternative, entweder die Eindampfanlage zu vergrössern, oder die Molke mit Umkehrosmose einzudicken. Die Umkehrosmose erschien mehr vorteilhaft. (3 Fälle)
- die Molke durfte nicht mehr in den Abfluss geleitet werden. Sie wird nun mittels Umkehrosmose zu 10% Trockensubstanzinhalt eingedickt und zusammen mit der Magermilch als Viehfutter returniert. (1 Fall)
- die Molke wird zu einer zentralen Eindampfanlage transportiert. Vorkonzentrieren mittels Umkehrosmose verringert den Kostenaufwand für Kühlung, Transport und Dampf in der zentralen Eindampfanlage. (1 Fall).

Der Kostenaufwand für eine RO-Anlage besteht hauptsächlich aus folgenden Posten:

- Kapital
- Membranen
- Reinigung

nom att förkoncentrera vasslen med omvänd osmos minskar kostnaderna för kylning, transport till och ångförbrukning vid centrala industninsanläggningen. (1 fall).

Kostnaderna för en RO-anläggning hänför sig huvudsakligen till följande poster:

kapital
membran
rengöring
arbetskostnad
elenergi
kylning.

I vissa fall sker pastörisering före eller efter RO-anläggningen. Pastöriseringen är kostsam men behandlas inte i detta arbete.

Kapitalkostnader

Fig. 4 visar investeringskostnaderna för de fem besökta RO-anläggningarna och ytterligare en just färdigställd PCI-anläggning. Priset är det som gällde vid köpet. Ingen hänsyn har tagits till att det skiljer 1.5 år mellan äldsta och yngsta anläggningen. Under denna tid har t. ex. engelska pundkursen fallit, vilket inverkar på priset av de i England tillverkade PCI-anläggningarna. Av fig. 4 framgår att investeringskostnaderna är ungefär lika stora för motsvarande DDS och PCI-anläggning.

De besökta mejerierna kalkylerade med avskrivningstider mellan 8 och 10 år och 8% räntesats, vilket ger en annuitet av ca. 15%. I de svenska mejerierna kalkyleras med omkring 5 års avskrivningstid, och 15–20% räntesats vad gäller nya anläggningar. Detta ger en annuitet närmare 30%. För de fortsatta beräkningarna användes en annuitet av 20%.

Membrankostnad

Kostnaderna vid membranbytet utgörs dels av arbetskostnad dels av membrankostnad. Som tidigare nämnts hade inga membranbyten skett i de anläggningar som besöktes, men kostnaderna går ändå att uppskatta ganska väl med uppgifter från membrantillverkarna. Kostnaderna ligger omkring 170 Skr./m² membran- yta varav arbetskostnaderna utgör mindre än 10%. Membranlivslängden har uppskattats till 1.5 år.

Rengöringskostnad förutom arbetskostnad

Rengöring sker en gång per dygn. Kostnaderna härrör sig från kemikalier, vatten, värme, elenergi och arbetskraft. Arbetskraften behandlas nedan. Övriga poster studerades noggrannt vid en DDS-anläggning och en PCI-anläggning båda med 56 m² membran- yta. I båda fallen erhöles rengöringskostnader exkl. arbetskraft till ca. 16 Skr./dygn. DDS skall dock börja använda ett nytt en-

Fig. 4. *Investeringskostnaden för RO-anläggningar för vasslekoncentrering som funktion av anläggningsstorlek.*
Fig. 4. *The investment costs of RO-plants as function of the plant size.*

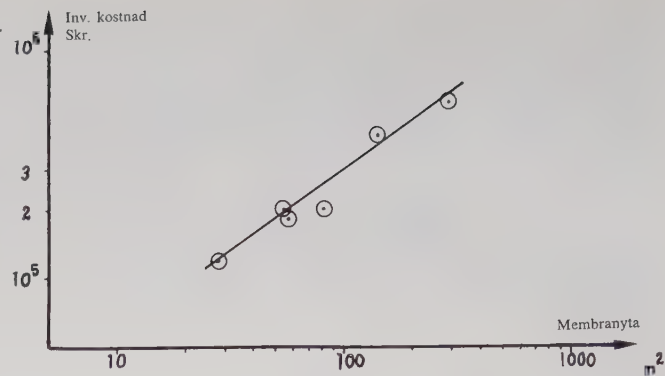


Abb. 4. *Investitionskosten für RO-Anlagen für Molkekonzentrierung als Funktion der Anlagengrösse.*

rating plant are decreased. (One case).

The costs of a RO-plant are mainly:

capital costs
membrane costs
cleaning costs
labour costs
electricity costs
cooling costs.

Sometimes the whey or whey concentrate is pasteurized. Pasteurization is costly but will not, however, be treated in this article.

Capital costs

Fig. 4 shows the investment costs of the five visited RO-plants and another newly-delivered PCI-plant. The shown investment costs are those the dairies had to pay for the RO-plants. No regards have been taken to the fact that the oldest plant is 1.5 year older than the youngest one. During this time for instance sterling has fallen, which influences on the investment costs of the English-made PCI-plants. Fig. 4 shows that the investment costs are about the same for comparable DDS- and PCI-plants.

The visited dairies calculated with a depreciation time between 8 and 10 years and 8% interest rate, which gives an annuity of about 15%. The Swedish dairies calculate with about 5 years depreciation time and 15–20% interest rate for new plants. This will give an annuity of about 30%. For the following calculations an annuity of 20% is used.

Membrane costs

The membranes costs are, on one hand the costs of the membrane, on the other labour costs. As mentioned earlier no change of membranes had been carried out at the visited plants. Yet it is possible to make rather good cost estimates from information from DDS and PCI. The costs are about 170 Swedish kronor/m² membrane area of which less than 10% is labour costs. The membrane life time is estimated at 1.5 years.

Löhne
Elektrische Energie
Kühlung.

In einigen Fällen wird die Molke vor oder nach der RO-Anlage pasteurisiert. Die Pasteurisierung ist kostspielig, wird aber hier nicht behandelt.

Kapitalaufwand

In Fig. 4 sind die Anlagekosten für die 5 besichtigten RO-Anlagen, sowie für eine gerade jetzt fertiggestellte PCI-Anlage dargestellt. Es sind diejenigen Preise angeführt, die beim Zeitpunkt der Anschaffung galten, die Zeitspanne von 1,5 Jahren zwischen der Anschaffung des jüngsten und ältesten Apparates wurde nicht berücksichtigt. So ist z.B. der Pfundkurs gesunken, was den Preis der in England hergestellten PCI-Anlage beeinflusst. Aus Fig. 4 geht hervor, dass die Anschaffungskosten für die entsprechenden DDS- und PCI-Anlagen ungefähr gleichgross sind.

Die besuchten Molkereien kalkulieren mit einer Abschreibungsdauer zwischen 8–10 Jahren, und einem Zinssatz von 8% was eine Annuität von ca. 15% ergibt. In den schwedischen Molkereien wird mit einer Abschreibungsdauer von 5 Jahren und einem Zinssatz von 15–20% für Neuanlagen kalkuliert. Dies ergibt eine Annuität von nahezu 30%. Für die weiteren Berechnungen wird mit einer Annuität von 20% gerechnet.

Kosten für Membranen

Die Kosten für Membrantausch setzen sich aus Arbeits- und Membrankosten zusammen. Wie schon früher erwähnt, wurden in den besuchten Anlagen bisher keine Membranen ausgetauscht, aber die Kosten können ziemlich verlässlich aufgrund von Angaben der Membranhersteller geschätzt werden. Die Kosten liegen ungefähr bei 170 Skr/m² Membranfläche wobei die Arbeitskosten weniger als 10% ausmachen. Die Lebenszeit der Membranen wurde auf 1,5 Jahren geschätzt.

zymtvättmedel som skall sänka rengöringskostnaderna. Här antages dock ovan nämnda kostnad och att den är direkt proportionell mot membranytan.

Arbetskostnad

Arbetskostnaden beräknas på olika sätt på olika mejerier. Ena gränsfallet är att RO-anläggningen belastas med arbetskostnad för en operatör under hela drifttiden för tillsyn. Andra gränsfallet är att RO-anläggningen inte belastas med någon arbetskostnad eftersom skötseln av den kunnat samordnas med övriga arbetsuppgifter så att ingen total arbetstidsökning har uppstått. Här beräknas arbetskostnaden efter den tid operatören är bunden vid RO-anläggningen, 1 tim för DDS-anläggning och 1.5 tim för PCI-anläggning. Timkostnaden antages 40 Skr. Arbetstiden är ganska oberoende av anläggningens storlek.

Elkostnad

Utgående från uppmätta värden av pumpens uppforderingshöjd och flöde samt antagen pumpverkningsgrad 0.8 fås pumpens elenergiförbrukning för en DDS-anläggning (satsvis) resp. PCI-anläggning till ca. 16 resp. ca. 4 kWh/m³ permeat. Elpriset antages 0.12 Skr./kWh.

Enl uppgift från DDS har vissa förbättringar gjorts så att energiförbrukningen blir sänkt till 11 kWh/m³ permeat i kommande satsvisa RO-anläggningar. I de följande beräkningarna antages dock energiförbrukningen till de här uppmätta 16 kWh/m³ permeat för DDS-anläggningar.

Kylkostnad

Vasslen i DDS-anläggningen måste kylas kontinuerligt p.g.a. den satsvisa processen vid låg temperatur. Kylningen utförs vanligen med isvatten. Om kapaciteten på befintligt isvattensystem är tillräckligt även för kylning i RO-anläggningen blir marginalkostnaderna härför i storleksordningen 5 kWh/m³ permeat à 0.12 Skr. Om isvattensystemet måste byggas ut blir kostnaderna för kylning betydligt större.

Dessutom måste varje m³ permeat i en DDS-anläggning ha kylts från 30°C till 10°C vilket inte behövs i en PCI-anläggning. Om denna kylning sker med isvatten åtgår ytterligare ca. 7 kWh/m³ permeat.

I de fortsatta beräkningarna antages det åtgå 5 kWh à 0.12 kr. per m³ permeat vid kylningen i en DDS-anläggning. Det bör poängteras att kylkostnaden i vissa fall kan bli betydligt högre.

PCI-anläggningen kräver ingen kylning.

Cleaning costs excluding labour

Cleaning is performed once a day. The cleaning costs arise from chemicals, water, steam, electrical energy and labour. The labour is treated below. The other items were closely examined at one DDS- and one PCI-plant both with 56 m² membrane area. In both cases the cleaning costs excluding labour were about 16 Skr/day. According to information from DDS, the DDS-plants will change their enzyme cleaning agent so the cleaning costs will be decreased. However for the following calculation the above mentioned cleaning costs are used and it is assumed that it is directly proportional to the membrane area.

Labour costs

The labour costs are calculated in different ways in different dairies. The borderlines are

1. The labour costs of the RO-plant are the costs of one operator for the whole operating time.
2. There are no labour costs, as the RO-plant operation has been coordinated to other tasks, so that the total working hours are not increased.

For the following calculations the labour costs are obtained from that time an operator has to serve the RO-plant. It is for a DDS- and a PCI-plant 1 hour/day and 1.5 hour/day respectively. The labour costs are assumed to be 40 Skr/hour. The working hours are relatively independent of the plant's size.

Electricity costs

The flow rate and discharge pressure of the feed pumps were measured. If the total efficiency of the pumps is 0.8 the energy consumption is about 16 and 4 kWh/m³ permeate for a DDS- and a PCI-plant respectively. The electricity cost is assumed to be 0.12 Skr/kWh.

According to information from DDS, certain improvements have been made so the energy consumption will decrease to 11 kWh/m³ permeate in new batchwise DDS-plants. However, for the following calculations the energy consumption is assumed to be 16 kWh/m³ permeate for DDS-plants.

Cooling costs

In the DDS-plant the whey has to be cooled continuously because of the batch-wise process at a low temperature. The cooling medium is usually ice water. If the capacity of the cooling system is enough even for the RO-plant, the marginal cost will be the cost of roughly 5 kWh/m³ permeate. If the cooling system has to

Reinigungskosten ohne Löhne

Die Apparatur wird einmal am Tage gereinigt. Die Kosten stammen von Chemikalien, Wasser, Wärme und der Arbeitskraft, die weiter unten behandelt wird. Die anderen Posten wurden eingehend bei einer DDS- und einer PCI-Anlage – beide mit 52 m² Membranfläche – studiert.

In beiden Fällen ergaben sich Reinigungskosten, excl. Arbeitskosten, von ungefähr 16 Skr/T. DDS plant die Anwendung eines neuen Enzymwaschmittels, das die Reinigungskosten herabsetzen wird. Hier wird doch mit den obenerwähnten Kosten gerechnet, und es wird angenommen, dass diese der Membranfläche proportional sind.

Arbeitskosten

Die Arbeitskosten werden in den verschiedenen Molkereien auf verschiedene Weise berechnet. In dem einen Grenzfall wird die RO-Anlage mit den Arbeitskosten für einen Operateur für die ganze Betriebszeit belastet. In dem anderen Grenzfall wird die RO-Anlage überhaupt nicht mit Arbeitskosten belastet, da deren Bedienung so mit den übrigen Betriebsaufgaben koordiniert wurde, dass die totale Arbeitszeit sich nicht verlängern würde. Hier wurden die Arbeitskosten auf der Grundlage der Zeit, die der Operateur bei der Apparatur verbringen muss, berechnet, u.z. 1 Stunde f.d. DDS-Anlage und 1,5 Stunden für die PCI-Anlage. Für eine Arbeitsstunde wurden 40 Skr. angesetzt. Die Arbeitszeit ist unabhängig von der Grösse der Apparatur.

Stromkosten

Von den gemessenen Werten für Förderhöhe und geförderten Menge ausgehend erhält man bei einem angenommenen Wirkungsgrad von 0,8 für die DDS-Anlage (satzweise) bezw. PCI-Anlage, einen Stromverbrauch von 16 bezw. 4 kWh/m³ Permeat. Als Strompreis wird 0,12 Skr/kWh angenommen.

Laut Angaben von DDS wurden durch einige Verbesserungen der Stromverbrauch in kommenden RO-anlagen auf 11 kWh/m³ Permeat herabgesetzt. In den folgenden Berechnungen wurde jedoch von den gemessenen 16 kWh/m³ Permeat ausgegangen.

Kosten für Kühlung

In einer DDS-Anlage muss die Molke aufgrund des satzweisen Prozesses kontinuierlich gekühlt werden. Gewöhnlich wird mit Eiswasser gekühlt. Falls die Kapazität des vorhandenen Eiswassersystemes auch für die Kühlung der RO-Anlage ausreicht, ergibt dies einen Aufwand von ungefähr 5 kWh/m³ Permeat à 0,12

Tabell 3. Uppskattade kostnader för RO-anläggningar med membranyta 140 m². Koncentreringsgrad 1:2, 250 driftsdagar/år.
 Table 3. Estimated costs for whey concentration RO-plants. Membrane area: 140 m². Concentration ratio: 1:2. Operating days/year: 250.
 Tabelle 3. Geschätzte Kosten für RO-Anlagen mit einer Membranfläche von 140 m². Konzentrierungsgrad 1:2, Betriebszeit 250 Tage/a.

Kostnadsslag	Kostnad Skr./år		
	DDS-anläggning	PCI-anläggning	
Kapital	76.000	76.000	Capital costs
Membran	15.900	15.900	Membrane costs
Rengöring	10.000	10.000	Cleaning costs
Arbete	10.000	15.000	Labour costs
El inkl. driftstid 7 tim/dygn	12.300	2.300	Electricity costs 7 hours/day
kylning			incl. cooling
driftstid 21 tim/dygn*	37.000	7.100	21 hours/day
Summa vid driftstid 7 tim/d	124.200	119.200	Sum at operating time 7 hours/day
Summa vid driftstid 21 tim/d*	148.900	124.000	Sum at operating time 21 hours/day*
	Costs Skr/year		
	DDS-plant	PCI-plant	

* PCI-anläggningen kommer förmodligen att belastas med ytterligare kyl- och värmekostnader (se texten nedan).
 * Additional costs for cooling and heating will probably debit the PCI-plant (see the text below).
 * Die PCI-Anlagen werden vermutlich mit weiteren Kühl- und Wärmekosten belastet werden. (s. weiterer Text).

Tabell 3 visar kostnaderna med ovanstående antaganden för koncentration av vassle i förhållande 1:2. Investeringskostnaden är beräknad efter den räta linjen i fig. 4.

Det framgår att kapitalkostnaden är den stora posten. Vid mindre anläggningar överväger den ännu mer. Härav följer att totalekonomin starkt beror av vilken annuitet köparen räknar med. Även arbetskostnaden är betydande. Arbetstiden går att minska starkt med ökad automatisering, men då ökar kapitalkostnaderna. En avvägning här emellan får ske i varje särskilt fall. Den stora skillnaden mellan DDS- och PCI-anläggningar är elkostnaderna. Detta visar sig speciellt vid hög ytnyttjandegrad av anläggningen. I praktiken förekommer dock sällan en jämn ström av färsk vassle i över 7 tim/dygn. Vasslen måste då kylas, mellanlagras och sedan värmas om en PCI-anläggning skall arbeta kontinuerligt under större delen av dygnet. Detta medför betydande tilläggskostnader, som dock inte behandlas här.

Fig. 5 visar kostnaderna per m³ avlägsnat vatten som funktion av anläggningsstorlek och driftstid per dygn. Kostnaderna är beräknade för en PCI-anläggning med ovan gjorda antaganden. Hade kostnaderna beräknats utgående från en DDS-anläggning hade kurvorna blivit nästan identiska. Maximal avvikelse i figuren skulle vara 1.9 Skr./m³.

Mängd permeat/dygn är proportionellt mot driftstid/dygn medan driftstiden endast påverkar elkostnaderna i kostnaderna. Eftersom elkostnaderna är mycket låga för en PCI-anläggning blir totalkostnad/m³ permeat så gott som omvänt proportionellt mot driftstid/dygn.

be enlarged the cooling costs will be considerably higher.

In addition each m³ permeate from a DDS-plant has to be cooled from 30 °C to 10°C which is not the case for a PCI-plant. If ice water is used for this cooling additional 7 kWh/m³ permeate is required.

For the following calculations, it is assumed that the cooling costs are 5 kWh/m³ permeate at 0.12 Skr/kWh for a DDS-plant. It must be accentuated that the cooling costs sometimes are considerably higher. The PCI-plants do not need cooling.

Table 3 shows the costs for whey concentration in a ratio of 1:2. The investment costs are calculated according to the straight line in fig. 4.

It appears that the capital costs are the highest. For smaller plants they dominate still more. Consequently the total economy is highly dependent on the annuity used. The labour costs too are considerable. It is possible to highly decrease the working hours by increased automation, but at the price of increased capital costs. The labour costs and capital costs will be balanced against each other in every special case. The big difference between DDS- and PCI-plants is the electricity costs. This is striking, especially at a large number of operating hours/day. Usually, a continuous flow of raw whey for more than 7 hours/day seldom occurs. The whey then has to be cooled, stored and then heated to 30 °C if a PCI-plant will operate continuously during a great part of the day. This will cause additional costs, which however not are treated in this work.

Fig. 5 shows the total cost/m³ removed water as a function of the RO-

Skr/kWh. Falls das Eiswassersystem ausgebaut werden muss, werden die Kosten für die Kühlung bedeutend grösser.

Ausserdem muss in einer DDS-Anlage jedes m³ Permeat von 30°C auf 10°C gekühlt werden, was in einer PCI-Anlage nicht notwendig ist. Wird diese Abkühlung mit Eiswasser erreicht, erfordert das weitere 7 kWh/m³ Permeat.

In den folgenden Berechnungen wird für eine DDS-Anlage ein Kühlaufwand von 5 kWh à 0,12 Skr/m³ Permeat angenommen. Es soll betont werden, dass in gewissen Fällen die Kosten für die Kühlung bedeutend höher sein können.

PCI-Anlagen erfordern keine Kühlung.

In der Tabelle Nr. 3 sind die Kosten für die Konzentrierung von Molke im Verhältnis 1:2 unter den oben angeführten Annahmen aufgezogen. Die Anlagekosten sind nach der Geraden auf Abb. 4 berechnet.

Es ergibt sich, dass die Kapitalkosten den grossen Posten darstellen. Bei kleineren Anlagen überwiegen sie noch mehr. Daraus folgt, dass die Totalökonomie in grossem Masse davon abhängt, mit welcher Annuität der Käufer rechnen kann. Der Lohnposten ist auch bedeutend. Es ist möglich, die Arbeitskosten durch Automation herabzusetzen, doch erhöht dies die Kapitalkosten. Hier muss in jedem Einzelfall eine ökonomische Erwägung entscheiden. Der grosse Unterschied zwischen PCI und DDS ist durch die Stromkosten bedingt. Dies tritt besonders bei hohem Ausnützungsgrad hervor. In der Praxis ergibt sich doch selten eine gleichmässige Frischmolkezufuhr für mehr als 7 Stunden/Tag. In dem Fall muss die Molke gekühlt, gelagert und wie-

Fig. 5. Totalkostnad som funktion av anläggningsstorlek och utnyttjandegrad vid koncentration av vassle med RO: 20 driftsdagar/lår: genomsnittligt permeatflux 250 l/m² tim. Ev kan ytterligare kyl- och värmekostnader tillkomma.

Koncentreringsgrad 1:2.

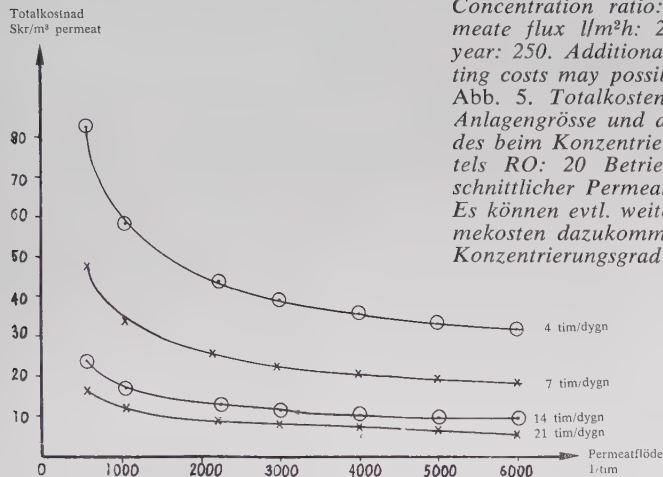
Fig. 5. The total cost of whey concentration by RO as a function of the plant's size and daily operating time.

Concentration ratio: 1:2. Average permeate flux 1/m²h: 20. Operating days/year: 250. Additional cooling- and heating costs may possibly arise.

Abb. 5. Totalkosten als Funktion der Anlagengröße und des Ausnutzungsgrades beim Konzentrieren von Molke mittels RO: 20 Betriebstage/Jahr, durchschnittlicher Permeatfluss 250 l/m²h.

Es können evtl. weitere Kühl- und Wärmekosten dazukommen.

Konzentrierungsgrad 1:2.



der aufgewärmt werden, wenn eine PCI-Anlage kontinuierlich mit einer höheren Stundenzahl pro Tag arbeiten soll. Dies ergibt bedeutende zusätzliche Kosten, die hier nicht behandelt werden.

Abb. 5 zeigt die Kosten pro m³ entferntes Wasser, als Funktion der Anlagengröße und der Betriebsdauer pro Tag. Die Kosten wurden für eine PCI-Anlage unter den obenangeführten Annahmen berechnet. Würden die Kosten für eine DDS-Anlage berechnet, so würden sich fast identische Kurven ergeben. Die maximale Abweichung in der Darstellung würde 1,9 Skr/m³ ergeben.

Die Permeatmenge/Tag ist proportionell zur Betriebszeit/Tag während die Betriebszeit nur die Stromkosten beeinflusst. Nachdem die Stromkosten für eine PCI-Anlage sehr niedrig sind, bewirkt dies, dass die Totalkosten/m³ Permeat so gut wie umgekehrt proportional zur Betriebszeit/Tag werden.

Schlussfolgerungen

Das Konzentrieren von Molke mit Umkehrosmose ist nun eine etablierte Methode. Die Apparathersteller sind DDS und PCI. Die gelieferten Apparate funktionieren nun nach Überwindung gewisser »Kinderkrankheiten« mit der Hilfsapparatur gut. Den grössten Anteil der Konzentrierungskosten machen die Kapitalkosten aus. Die diskontinuierlichen DDS-Anlagen erfordern weniger Arbeitskraft, aber mehr Strom, als die kontinuierlichen PCI-Anlagen. Es ist wichtig, dass das Bedienungspersonal gut instruiert wird, wie die Anlage betrieben werden soll, sodass die Lebensdauer der Membranen maximal wird.

Slutsatser

Koncentrering av vassle med omvänd osmos är nu en etablerad metod. Tillverkare av utrustning för detta är huvudsakligen DDS och PCI. Levererade anläggningar fungerar nu bra efter vissa initiala problem med kringutrustningen. Den största delen av kostnaderna för koncentrationen utgörs av kapitalkostnader. Satsvisa DDS-anläggningar kräver mindre arbetskraft men mera elenergi än kontinuerliga PCI-anläggningar. Det är viktigt att den personal som sköter RO-anläggningen är väl instruerad hur anläggningen skall skötas så att membranlivstiden maximeras.

plant's size and daily operating time. The total cost is calculated from assumptions given above for a PCI-plant. The curves for a DDS-plant would be nearly identical with them in fig. 5. The maximum difference would be 1.9 Skr/m³.

The permeate volume/day is directly proportional to operating time/day. The only costs which are dependent on operating time/day are the electricity costs, and as they are very small for a PCI-plant, the total cost/m³ permeate is almost inversely proportional to operating time/day.

Conclusions

Whey concentration by reverse osmosis is now an established method. Manufacturers of existing plants are mainly DDS and PCI. The plants in operation are now working well after some initial problems with the auxiliary equipment. The greatest part of the total cost of concentration is the capital cost. Batch-wise DDS-plants require less labour but more electrical energy than continuous PCI-plants. It is important that the operators of the RO-plant are well informed about how to run the plant, so the membrane-life is maximized.

plants which use the lignosulfonates for the production of vanillin.

In some applications, the lignosulfonates need not be pure, and it is then sufficient to evaporate and dry the SSL. For example, when lignosulfonates are used as a binder for animal feed, the sugars following the lignosulfonates have nutritional value. In other applications, a rather pure lignosulfonate fraction is required, and is obtained in practice either by sugar fermentation described above or by ultrafiltration (UF).

However, the lignosulfonates in the SSL are not a homogeneous fraction. It is rather difficult to measure their molecular weight and it has been reported that they have a molecular weight distribution that lies between 5000 and over 100000 Daltons (Collins et al. (2)). There are some applications for which the product quality is much improved if only a high molecular fraction of the lignosulfonates is used, for example for its use as dispersants and adhesives. Consequently, this high molecular fraction is of great value, and the only suitable way to separate it is by ultrafiltration.

ULTRAFILTRATION MODULES

The SSL contains a great deal of fibers, most of which can be separated by using a prefilter but, as they are very thin a small amount will always pass the prefilter under the prevailing practical conditions. Therefore, it is important that the membrane module is so constructed that it will not be clogged by these fibers. This would occur with the compact and cheap spiral wound and externally pressurized hollow fine fiber modules. It is the feed side spacer that encumbers the passage for the fibers in the spiral wound modules. Tubular and "plate and frame" systems are not as sensitive to solid particles, and both systems have been used for several years with SSL without problems. Internally pressurized hollow fibers with an inner diameter of about 1 mm can eventually be used for this application, but they are more susceptible to clogging than tubular and "plate and frame" systems.

MEMBRANES

The economy of lignosulfonate concentration by ultrafiltration is very much dependent on the permeate flux of the membrane. The major cost items for an UF-plant are capital costs, membrane costs, and electrical costs, and all

these are nearly inversely proportional to the permeate flux for a given large capacity. The permeate flux increases with the temperature since the viscosity of the solution decreases and the diffusivities of the species increase. The temperature of the fresh SSL is usually about 90°C and if the membrane could withstand this temperature, no cooling device would be needed, which is a further advantage. The rejection properties of the membrane generally decrease with increasing temperature, but the higher flux achieved usually more than compensates for the additional lignosulfonate losses.

So far, it is only dynamic membranes on inorganic supports which continuously withstand as high a temperature as 90°C. Collins et al. (3) and Bansal, Wiley (4) tested dynamic membranes on Westinghouse "sand log" modules for SSL-fractionation, but they neither got higher flux nor better performance with these membranes at 90°C compared to other commercial organic membranes which can withstand 70-80°C. Since the capital cost for a given membrane area is at present higher for dynamic membranes on inorganic supports than for commercial organic membranes, the former are not so interesting for commercial use.

Bansal, Wiley (1,4) and Collins et al. (3) have also tested cellulose-acetate membranes for SSL-fractionation. Usually these membranes have a rather short lifetime at temperatures above 30°C and pH about 3, but these authors found that they functioned with SSL at 55°C for over 1000 hours without deterioration. This is probably because the lignosulfonates form a dynamic layer on the real membrane, and if the real membrane is not too tight, it is the dynamic layer which determines the flux and rejection data. The dynamic layer may also protect the cellulose-acetate membrane from degradation.

To maintain a high permeate flux, the membrane needs regular cleaning and this is performed in the cheapest and fastest way by using a warm alkaline solution. However, cellulose-acetate membranes can't withstand this cleaning method and no manufacturer can warrant a lifetime of even half a year for these membranes for SSL fractionation at 55°C. Accordingly, cellulose-acetate membranes are not used on an industrial scale for this application despite their low cost per unit area.

De Danske Sukkerfabrikker (DDS) uses polysulfone membranes for SSL fractionation. These membranes can withstand 80°C continuously, but are 2-3 times more expensive than cellulose-

acetate membranes per unit area. Other UF-plant manufacturers interested in SSL fractionation use membranes which are claimed to withstand a temperature of at least 70°C.

As mentioned above, the lignosulfonates will form a dynamic layer on the membrane. Since the lignosulfonates are strongly hydrophilic, their water permeability is rather high. This means that the dynamic layer would not directly decrease the permeate flux to any considerable extent. However, this layer greatly reduces the back diffusion of molecules rejected by the membrane. If there are other slightly hydrophilic molecules which are rejected by the membrane, then the dynamic layer will cause an increased concentration of these molecules near the membrane surface. This will strongly decrease the permeate flux. Bar-Sinai, Wayman (5) found that for sodium-base SSL, sugar was rejected by membranes with a nominal cut-off of 10000 and less. According to Haagensen (6), the permeate flux for the SSL is much less using DDS polysulfone membrane with a cut-off of 10000 than with other DDS polysulfone membranes with nominal cut-off of 25000 and 65000. This indicates that to get a high permeate flux in SSL fractionation, the membrane must be coarse enough to let not strongly hydrophilic molecules through. Inevitably, there must be some loss of low molecular weight lignosulfonates.

SYSTEM PERFORMANCE

The variation of the permeate flux and the lignosulfonate purity in the concentrate with the conversion ratio (Permeate volume/Feed volume) in a typical case is shown in Fig. 1. Since the absolute value of the permeate flux depends on many factors, e.g. membrane, temperature, type of SSL and its dry solids content, no absolute values are used on the flux scale. Moreover, it is evident from the figure that there is an upper limit to the lignosulfonate purity attained by straight forward ultrafiltration and furthermore the permeate flux decreases rapidly with increasing conversion ratio.

To obtain a high given lignosulfonate purity the SSL is first ultrafiltered to a suitable conversion ratio. The ultrafiltration is then continued by adding water at constant solids concentration of the concentrate until the desired lignosulfonate purity is obtained. This process is called diafiltration (DF). If diafiltration is started early the permeate flux will be high all the time, and a large amount of fresh water must be added to obtain a given

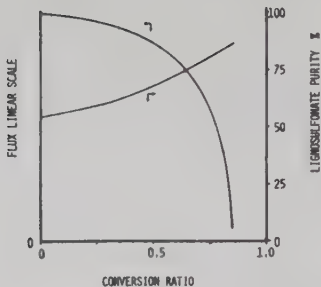


Fig. 1 Typical values of the lignosulfonate purity and relative permeate flux by SSL ultrafiltration.

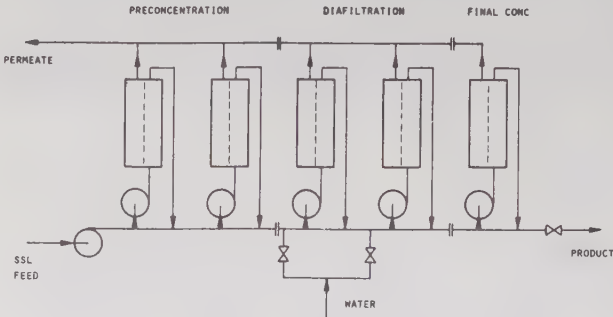


Fig. 2 Continuous UF/DF-plant.

TABLE 1. PROCESSING - AND COST FIGURES FROM UF/DF OF SSL.

Coarse UF membrane	Ultra-filtration	Conc.	Diafiltration	Conc.
SSL	1176 m ²	C _{PC}	942 m ²	C _{DF}
	Permeate P _{PC}		Permeate P _{DF}	

MASS BALANCE-CONCENTRATIONS					
	SSL	P _{PC}	C _{PC}	P _{DF}	C _{DF}
Total mass t/h	120	93.3	26.7	35.5	19.2
Dry solids content wt-%	10.0	5.7	25.0	5.3	25.0
Dry solid mass t/h	12	5.33	6.67	1.87	4.8

APPROXIMATE COST FIGURES			
	Ultrafiltr.	Diafiltr.	Total
Investment \$	1.4·10 ⁶	1.1·10 ⁶	2.5·10 ⁶
Electricity consumpt. kW	450	350	800
Membrane exchange \$/year	108·10 ³	85·10 ³	193·10 ³

COMPOSITION OF SOLIDS					
Lignosulfonate t-% of solids	6.6-55	1.13-21.2	5.47-82	1.01-54	4.46-92.9
Sugars and acids t-% of solids	3.96-33	3.08-57.8	0.86-13.2	0.63-33.7	0.25-5.2
Inorganics t-% of solids	1.44-12	1.12-21.0	0.32-4.8	0.23-12.3	0.09-1.9

CAPITAL COST	\$/YEAR	750·10 ³
ELECTRICITY COST	\$/YEAR	192·10 ³
MEMBRANE COST	\$/YEAR	193·10 ³
SUM	\$/YEAR	1.135·10 ⁶
DRY PRODUCT	KG/YEAR	38.4·10 ⁶
COST/PRODUCT	\$/KG	0.03

TABLE 2. MAJOR COSTS FOR THE COMBINED UF AND DF PLANT

ANNUITY	30 %
ELECTRICITY COST \$/tHJ	0.0083
OPERATING TIME H/YEAR	8000

lignosulfonate purity. The membrane area thus required will be large. On the other hand, if diafiltration is started late, only a small amount of fresh water is required to obtain the same lignosulfonate purity. But as the permeate flux in this case is low, a large membrane area is still required. Consequently, there is an optimum of the conversion ratio at which diafiltration should be started in order to minimize the membrane area required. For digester-strength SSL, this conversion ratio optimum is about 0.75-0.8.

A batch system is suitable for both testing and small-scale production. It is simple and easily controlled, but its electrical power consumption is high. For higher capacities, continuous multistage systems are used, as illustrated in fig. 2. These units can be operated fully automatically.

ECONOMY

Claussen (7) presented some processing and cost data for combined ultrafiltration and diafiltration of a typical digester-strength SSL. These are reproduced in table 1 with a cost update (April -79).

Since a modular arrangement with constant unit size is used for the UF/DF-plant, the economy is not so dependent on the plant size. For instance, if the feed flow is one fourth of that in table 1, the costs would also be very near one fourth of those given in the same table.

The end product is a concentrate of 25% dry solids content with 93% lignosulfonate purity. About 32% of the lignosulfonates in the feed are lost to the permeate, so this plant alone is not enough for pollution control. It is the low molecular weight fraction which is lost. However, the objective is to get a valuable lignosulfonate fraction.

Usually the lignosulfonate will be shipped as a dry powder, and this is attained by evaporation and spray-drying. Thus it is usually not economical to concentrate to more than about 25% dry solids content by ultrafiltration, because of the low permeate flux at high solids content.

Table 2 shows that for the UF/DF-plant it is the capital cost which dominates the economy. The range of this cost will of course depend on the annuity chosen. The total cost here will be about 0.03 \$/kg dry product. The costs for further concentration by evaporation and spray-drying to get the product in powder form

will be of the same magnitude. The total cost to get a powder of high lignosulfonate purity from digester-strength SSL will thus be less than 0.10 \$/kg. Haagensen (6) claims that this powder product today is sold for about 0.4 \$/kg. This would mean that this process is quite profitable, even for mills that use a chemical recovery system.

The market today is, however limited and much work is now carried out to find new qualified applications for the pure lignosulfonate fraction.

DILUTED SOLUTIONS

The cost of producing the high purity lignosulfonate powder is not very dependent on SSL feed concentration. If the concentration is low, a larger membrane area per kg lignosulfonate is required for the UF-stage. On the other hand, since the permeate flux is very high for low concentrations, the extra membrane area required will be comparatively small. Moreover, the conversion ratio of the UF-stage will be higher at lower feed concentrations, thus a smaller amount of diafiltration water is required to get the same lignosulfonate purity. Consequently, the membrane area required in the DF-stage is then smaller.

The reason for using diafiltration is mainly to reduce the sugar content. As seen in table 1, the costs of the DF-stage are almost as high as those of the UF-stage. Besides, by diafiltration the permeate volume will be increased, and thus the cost for further treatment of the permeate will increase. If the SSL is first fermented so that most of the sugar is removed, then a sufficiently pure lignosulfonate fraction may be obtained without diafiltration. However, during the fermentation process some materials are formed which negatively influence the permeate flux. This may be overcome by proper pretreatment after the fermentation process.

COMMERCIAL UF/DF-PLANTS

The first commercial plant for SSL fractionation by UF/DF was started in Norway in 1975, and it has worked continuously since then. Its capacity is about 500 kg/h dry lignosulfonate powder. A similar plant with the same capacity is also in operation in the United States. Several pilot plants are in use in different countries.

Claussen (7) claims that all acid and bisul-

fite SSL's, alcohol and protein-fermented liquors may be treated by ultrafiltration and diafiltration.

CONCLUSION

By a combination of ultrafiltration and diafiltration of SSL, a valuable high molecular fraction of lignosulfonates is obtained. Membranes used for this application should have nominal cut off values over 10000, and the membrane module design must not be susceptible to clogging.

The market for purified lignosulfonates is limited today, and much research work is now carried out to find new qualified applications for this product.

The cost to produce a highly valuable lignosulfonate fraction of about 25% dry solids content by using a large scale UF/DF-plant amounts to about 0.03 \$/kg dry product.

ACKNOWLEDGEMENT

I would like to express my gratitude to Dr. Kaj Forss at the Finnish Pulp and Paper Research Institute, Dr. Ingvar Johansson at the Swedish Forest Products Research Laboratory, Ulf Haagensen and Per Holm Claussen, DDS and David Webster, PCI, for valuable information and Dr. Gharib Aly, the Lund Institute of Technology, for linguistic assistance.

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Coden: LUTKDH/(TKKA-3006)/1-119/(1980)

MASS TRANSFER IN TURBULENT DUCT FLOW

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Dokumentnamn
Report
Utgivningsdatum
November 1980

Dokumentbeteckning
LUTKDH/(TKKA-3006)
/1-119/(1980)
~~Ärendebeteckning~~

Dokumenttitel och undertitel

Mass Transfer in Turbulent Duct Flow

Referat (sammmandrag)

A critical examination of published correlations for the mass transfer coefficient at fully developed turbulent flow in circular ducts with smooth nonpermeable walls and constant physical properties at $Sc > 0.5$ is made. For the high Schmidt numbers encountered in reverse osmosis and ultrafiltration ($Sc > 500$), it is investigated for turbulent flow how the mass transfer coefficient depends on rough walls, the entrance region, turbulent Reynolds numbers below 10^4 , noncylindrical geometry, variable physical properties and permeable walls.

The solute rejection of the membrane decreases with increasing pressure difference if the permeate flux is greater than the zero flux limiting mass transfer coefficient. In ultrafiltration at turbulent flow the energy consumption per unit volume permeate is only very slightly dependent on the hydraulic diameter of the duct.

Referat skrivet av
P.E.
Förslag till ytterligare nyckelord

Klassifikationssystem och -klass(er)

Indextermer (ange källa)

Omfång
125 pages
Språk
English
Sekretessuppgifter

Övriga bibliografiska uppgifter

ISSN ISBN

Dokumentet kan erhållas från
Avd för Kemisk Apparatteknik
Kemicentrum, Box 740
220 07 LUND, Sweden

Mottagarens uppgifter

Pris

ABSTRACT

In turbulent duct flow whirls are generated in the region $5 < y^+ < 70$ and penetrate to the wall region and the core region. The whirls are irregular in time and space, hence the local concentration and mass transfer rate vary in time. It is impossible to analytically predict these time fluctuations. However some models give a realistic picture of the mass transfer phenomenon. In the core region the molecular mass transfer rate is negligible at $Sc > 1$ and the Reynolds analogy is applicable. In a transition region there is rapid whirl exchange and there the unsteady penetration model is applicable. In the wall region the whirl penetration frequency is very low and there mainly intermittent stationary boundary layer flow prevails. For fully developed flow in circular ducts with smooth non-permeable walls at $Re > 10^4$ and constant physical properties, Hughmark derived a reliable correlation for the mass transfer coefficient

$$\frac{1}{k} = \frac{1}{\frac{u^*}{1.6 \cdot Sc} + 0.047 \cdot u^* \cdot Sc^{-\frac{2}{3}}} + \frac{1}{\frac{u^*}{33 \cdot Sc} + 0.062 \cdot u^* \cdot Sc^{-\frac{1}{2}}} + \frac{1}{7.16 \frac{D}{d_h} + f \cdot u_b}$$

This correlation is rather inconvenient to use and at $0.7 < Sc < 300$ the correlations of Petukhov and Notter-Sleicher give about the same result

$$k = \frac{\frac{f}{2} \cdot u_b}{1.07 + 12.7 \cdot \sqrt{f/2} \cdot (Sc^{\frac{2}{3}} - 1)} \quad 0.7 < Sc < 300 \quad (\text{Petukhov})$$

$$k = \frac{D}{d_h} \cdot (5 + 0.015 \cdot Re^a \cdot Sc^b) \quad 0.7 < Sc < 300 \quad (\text{Notter and Sleicher})$$

$$a = 0.88 - \frac{0.24}{4 + Sc} \quad b = \frac{1}{3} + 0.5 \cdot \exp(-0.6 \cdot Sc)$$

At $Sc > 700$ the correlation of Shaw and Hanratty is the most reliable,

$$k = 0.0889 \cdot \sqrt{f/2} \cdot u_b \cdot Sc^{-0.704} \quad (\text{Shaw and Hanratty})$$

At $300 < Sc < 700$ the above correlations overpredict and underpredict respectively the mass transfer coefficient, so there the correlations of Deissler or Ruckenstein are recommended

$$k = 0.112 \sqrt{f/2} \cdot u_b \cdot Sc^{-\frac{3}{4}} \quad 300 < Sc < 700 \quad (\text{Deissler})$$

$$k = 0.074 \sqrt{f/2} \cdot u_b \cdot Sc^{-\frac{2}{3}} \quad 300 < Sc < 700 \quad (\text{Ruckenstein})$$

At high Schmidt numbers the concentration boundary layer is so thin that within it the shear stress can be considered as constant, and in these cases it is investigated how the mass transfer coefficient depends on wall roughness, entrance region, turbulent flow at $Re < 10^4$, noncylindrical geometry, variable physical properties and permeable walls.

Only that kind of roughness which itself does not constitute any mass transfer resistance is considered. Roughness may increase the mass transfer coefficient more than three-fold. For dimensionless roughness height above ten ($e^+ > 10$) the correlation for the mass transfer coefficient is

$$k = \text{const.} \cdot \sqrt{f/2} \cdot u_b \cdot (e^+)^{-0.24} \cdot Sc^{-0.59}$$

where the value of the constant depends on the geometry of the roughness elements and is in the order of 0.2.

With fully developed velocity profile in the entrance region, the quotient $\bar{k}/k_\infty$ is practically only dependent on x^+ , $1 + \frac{300}{x^+} < \frac{\bar{k}}{k_\infty} < 1 + \frac{500}{x^+}$. A developing velocity profile will decrease the mass transfer coefficient. Entry flow disturbances generate extra turbulence in the entrance region and increase the mass transfer coefficient, but more experimental investigations are needed to determine this effect quantitatively.

At $Sc > 2000$ the correlations for the mass transfer coefficient valid at $Re > 10^4$ are also valid at turbulent Reynolds numbers below 10^4 .

If the same correlations as for circular duct flow with the hydraulic diameter as the characteristic length are used to calculate the friction factor and the mass transfer coefficient at $Sc > 500$, the resulting error in the mass transfer coefficient will be less than 5% for rectangular duct flow at $Re > 10^4$, less than 3% for equilateral triangular duct flow at $Re > 5 \cdot 10^4$ and less than 7% for concentric cylindrical annular duct flow at $Re > 10^4$ and $r_i/r_o > 0.5$.

For variable physical properties and $Sc_w > Sc_b$ the correlation of Shaw and Hanratty is recommended, with the friction factor calculated from the bulk properties and the Schmidt number from the wall properties.

If the eddy diffusivity is independent of the transverse velocity through the wall (v_w) the mass transfer coefficient will be $k = v_w(1 - \exp(-v_w/k(v_w=0)))^{-1}$, which is the same result as that obtained by the film theory. This equation has been experimentally verified by others.

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1. INTRODUCTION

In laminar flow single molecules randomly cross the stream lines, which is called molecular transport or diffusion. The mathematical model of this object is well-known and can be found e.g. in Bird et al. (1960, chap. 3 and 18). Here, for simplicity, the two-dimensional case with rectangular coordinates is presented.

$$-\frac{\partial \rho}{\partial t} = \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v_y) \quad \text{Eq. of continuity}$$

$$-\rho \frac{\partial u}{\partial t} = \rho u \frac{\partial u}{\partial x} + \rho v_y \frac{\partial u}{\partial y} + \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} - \rho g_x \quad \begin{array}{l} \text{x-component of the} \\ \text{Eq. of motion} \end{array}$$

$$-\frac{\partial c}{\partial t} = \frac{\partial j_x}{\partial x} + \frac{\partial j_y}{\partial y} \quad \begin{array}{l} \text{Eq. of continuity for the solute} \\ \text{in a binary mixture without chem-} \\ \text{ical reaction} \end{array}$$

τ_{ij} is the shear stress exerted in the j -direction on a fluid surface of constant i by the fluid in the region of lesser i .

These equations are sometimes analytically solvable.

In turbulent flow mass and momentum are transferred by both molecular and turbulent transport. Turbulent transport is the process when lumps of fluid consisting of many molecules deviate from the local average flow direction or velocity. These lumps are also called whirls or eddies and they exist in a broad range of sizes. The mathematical model of turbulent flow is much more complicated than for the laminar one. For an incompressible fluid in two-dimensional flow and rectangular coordinates the equations are

$$0 = \frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v_y) \quad \text{Eq. of continuity}$$

$$-\rho \frac{\partial u}{\partial t} = \rho u \frac{\partial u}{\partial x} + \rho v_y \frac{\partial u}{\partial y} + \frac{\partial P}{\partial x} + \frac{\partial \tau_{xx}^{(m)}}{\partial x} + \frac{\partial \tau_{yx}^{(m)}}{\partial y} + \frac{\partial \tau_{xx}^{(t)}}{\partial x} + \frac{\partial \tau_{yx}^{(t)}}{\partial y} - \rho g_x$$

x-component of the Eq. of motion

$$-\frac{\partial c}{\partial t} = \frac{\partial j_x^{(m)}}{\partial x} + \frac{\partial j_y^{(m)}}{\partial y} + \frac{\partial j_x^{(t)}}{\partial x} + \frac{\partial j_y^{(t)}}{\partial y}$$

Eq. of continuity for the solute in a binary mixture without chemical reaction.

In turbulent flow the quantities are time dependent. However, here and henceforth the quantities represent their time average value, unless otherwise stated.

These equations can also be found in Bird et al. (1960, chapter 5 and 20). The laminar and turbulent mathematical models for incompressible flow deviate from each other in the terms containing $\tau^{(t)}$ and $j^{(t)}$. These terms are the momentum and mass respectively which are transferred by the turbulent eddies. The problem is that at present no method exists to calculate $\tau^{(t)}$ and $j^{(t)}$ a priori. Therefore great efforts have been made to produce correlations for the mass and momentum transfer in turbulent flow. Physical models, empirical correlations and combinations of them both have been used.

This work considers mass and momentum transfer for binary solutions in turbulent flow. For colloidal suspensions, solid particles, emulsions etc., the reader is referred to Porter (1972). This work can be divided in two parts. The first part (chapter 1-21) is mainly a critical literature survey and considers the mass transfer at $Sc > 1$ and the following conditions.

- smooth walls
- fully developed turbulent flow in circular ducts
- $Re > 10^4$
- constant physical properties
- nonpermeable walls

First the present-day knowledge of the turbulence phenomena near a solid wall is briefly presented. Then published experimental data for mass and heat transfer are presented. Then different models for mass transfer in turbulent duct flow are considered, which forms the main content of the first part of this work. These models and their degree of consistency with experimental data contribute significantly to the understanding of the turbulence phenomena. In the last section conclusions, empirical correlations and final recommendations are presented.

The second part of this work (chapter 22-27) considers mass transfer at high Schmidt numbers ($Sc > 500$). At high Schmidt numbers the concentration boundary layer is thin enough to let both the shear stress and the transverse mass flux be practically constant within it. Unless otherwise stated, in this work the boundary layer is the layer near the wall where 98% of change from the wall value to the bulk value of respective quantity occurs. It is investigated how the mass transfer coefficient is dependent on rough walls, the entrance region, turbulent Reynolds numbers below 10^4 , noncylindrical geometry, variable physical properties and permeable walls. For all these conditions theoretical analyses are carried out and compared with published experimental data.

As mass and heat transfer are analogous, this work is also applicable to heat transfer. In Table 1-1 the analogous quantities are shown.

<u>Mass Transfer</u>		<u>Heat Transfer</u>	
solute concentration	$c \text{ kg/m}^3$	energy concentration	$\rho c_p T \text{ J/m}^3$
mass diffusivity	$D \text{ m}^2/\text{s}$	thermal diffusivity	$\alpha \text{ m}^2/\text{s}$
mass transfer coefficient	$k \text{ m/s}$	heat transfer coefficient	$h \text{ m/s}$
Schmidt number	$Sc = \frac{\nu}{D}$	Prandtl number	$Pr = \frac{\nu}{\alpha}$
Sherwood number	$Sh = \frac{k d_h}{D}$	Nusselt number	$Nu = \frac{h d_h}{\alpha}$

Table 1-1. Analogous quantities at mass and heat transfer

Most of the models presented in this work are derived for heat transfer, but here they are converted to the analogous mass transfer.

2. TURBULENCE PHENOMENA NEAR A SOLID WALL

Corino and Brodkey (1969) studied how colloidal particles (average diameter $0.6 \mu\text{m}$) in a liquid behaved near the wall in turbulent circular duct flow. They found that the eddies were generated in the region $5 < y^+ < 70$ (generation region) with maximum at $7 < y^+ < 30$. $y^+ = y \cdot u^* / \nu$ is the dimensionless distance from the wall. From the generation region eddies penetrate in all directions. The turbulent flow is three-dimensional, i.e. the eddies move not only in radial and axial directions. Those eddies which penetrate the core region penetrate a long distance before mixing with the surrounding fluid. The mixing is then performed relatively calmly. In the generation region chaotic conditions prevail. Eddies come into collision with each other, which generates smaller eddies. Those eddies which penetrate the wall region ($y^+ < 5$) have a relatively low radial velocity, so they calmly push the surrounding fluid out of its place. The frequency of these eddies decreases with decreasing distance to the wall. When the eddies penetrate the wall region, the wall region axial velocity temporarily increases. Then small rapid irregular bursts of eddies are generated in the outer wall region and they penetrate to the generation region. In the generation region more eddies are generated when small low axial velocity eddies from the wall region come into collision with high axial velocity fluid.

Popovich and Hummel (1967) used a flash photolysis method to study the velocity field in a liquid near the wall in turbulent square duct flow. In their experiments they always found a linear velocity gradient for $y^+ < 1.6 \pm 0.4$, but the slope of the gradient changed with time. Then the longer the distance from the wall the more often turbulent flow prevailed and for $y^+ > 35$ they always found turbulent flow.

It is evident from both works described above that the velocity field near the wall varies much through time, and that the eddy frequency decreases with decreasing distance to the wall. Near the wall the shear stress is practically constant. The eddy frequency (n) near the wall then is a function of only the wall shear stress, the physical properties of the fluid and the distance from the wall.

$$n = n(\tau_w, \nu, \rho, y)$$

Buckingham's π -theorems give

$$n^+ \equiv \frac{n}{\tau_w / \rho \nu} = n(y^+)$$

$\sqrt{\frac{\tau_w}{\rho}}$ has the dimension [m/s] in SI-units and is called the friction velocity (u^*).

$$\frac{n}{u^{*2} / \nu} = n(y^+)$$

This means that at a constant dimensionless distance from the wall, the eddy frequency is proportional to u^{*2} / ν . This is confirmed by the above mentioned references.

For those eddies which do not reach the wall, mass and momentum exchange with the wall are achieved by molecular diffusion. The lower the molecular diffusivity the lower the response at the wall by those eddies which do not reach the wall. Thus the local mass transfer fluctuation frequency will decrease with decreasing mass diffusivity, i.e. increasing Schmidt number. On the other hand, as the mass transfer fluctuation frequency decreases, the amplitude will increase relatively. This is because those eddies which penetrate very near the wall affect the mass transfer relatively more the lower the mass diffusivity.

Shaw and Hanratty (1977b), used an electrochemical method to measure the fluctuations of the mass transfer to the wall for turbulent liquid flow in a circular duct. They obtained that the mass transfer fluctuations were dependent on the Schmidt number according to

$$\overline{n_w^+} \approx 0.022 \cdot Sc^{-0.41} \quad 700 < Sc < 17\,000 \quad (2-1)$$

$\overline{n_w^+}$ = mean dimensionless frequency weighted by the importance to the mass transfer to the wall.

Equation (2-1) is probably valid also for $Sc > 17\,000$ but not for low Schmidt numbers. For low Schmidt numbers even eddies in the generation region are of direct importance to the mass transfer to the wall, and there the eddy frequency does not increase with increasing wall distance.

Table 2-1 shows the influence of Schmidt number on $\overline{n}_w^+$ according to Eqs. (2-1 and 2) proposed below.

Sc	1	10	100	500	10^3	10^4
$\overline{n}_w^+ \cdot 10^3$ [Eq. (2-1)]	22	8.6	3.3	1.7	1.3	0.5
$\overline{n}_w^+ \cdot 10^3$ [Eq. (2-2)]	5.0	3.4	2.3	1.7		

Table 2-1. $\overline{n}_w^+$ according to Eqs. (2-1 and 2)

Other published results for the mass transfer fluctuations are presented in Table 2-2.

Author	Measured fluctuation quantity	$\overline{n}_w^+ \cdot 10^3$
Dworak and Wendt (1977)	liquid mass transfer, $Sc \approx 10^3$	0.82
Mitchell (Meek (1972))	liquid mass transfer	1.1
Meek and Baer (1979)	liquid heat transfer	1.1-2.8
Meek and Baer (1970)	pressure (air)	1.6-5.9
Corino and Brodkey (1969)	visually	4.4
Schraub et al. (Meek (1972))	visually	5.1
Sirkar and Hanratty (Hughmark 1971b)	shear stress	9.1

Table 2-2. Empirical wall fluctuation data

Meek and Baer (1970) measured, first the pressure fluctuations at the wall for turbulent air flow in a circular duct and second the wall temperature fluctuations for turbulent liquid flow in a circular duct. Meek (1972) presented published data of other authors about pressure, heat transfer and mass transfer fluctuations near the wall. Dworak and Wendt (1977) used an electrochemical method to measure the fluctuations of the mass transfer to the wall for turbulent liquid flow in a square duct.

Experiments where the eddy frequency is visually studied or determined by measuring pressure or shear stress fluctuations, correspond to mass transfer fluctuations at $Sc = 1$.

By comparing Table 2-1 and Table 2-2 it seems that Eq. (2-1) is not valid for $Sc=1$ but is probably valid for high Schmidt numbers. In later chapters we need an expression for $\overline{n}_w^+$ at low Schmidt numbers. This expression shall give $\overline{n}_w^+ \cong 0.005$ at $Sc = 1$ and about the same value as Eq. (2-1) at $Sc \cong 500$. Arbitrarily the expression is chosen as

$$\overline{n}_w^+ = 0.005 \cdot Sc^{-0.172} \qquad 1 < Sc < 500 \qquad (2-2)$$

Conclusion

Experimental investigations have shown that eddies are generated in a region $5 < y^+ < 70$. From this region eddies three-dimensionally penetrate the wall region, $y^+ < 5$. Within the wall region the eddy frequency decreases with decreasing distance to the wall.

3. EXPERIMENTAL MASS AND HEAT TRANSFER DATA

Diagram 1 shows experimental mass and heat transfer data at $Re = 10^4$. The diagram is used for comparing the different models in the following chapters with experimental data. Of course all these data were not available at the time when most of the models described below were produced. Most of the data at $Sc > 10^3$ are from the 1970's. Data at $Sc < 500$ are from heat transfer experiments.

Malina and Sparrow (1964) show that the Nusselt number at constant bulk Reynolds and Prandtl number varies with the temperature difference between the bulk and the wall. This is because of the temperature dependence of the physical properties. The higher the Prandtl numbers the greater influence of the temperature difference on the Nusselt number when heating. The data in diagram 1 symbolized with Δ , $\diamond$ and $\bullet$ are not extrapolated to zero temperature difference, which might cause too high values of the plotted Sherwood numbers, especially at high Prandtl numbers. Notter and Sleicher (1971) and Berger and Hau (1977) further describe the problems of obtaining reliable mass and heat transfer data.

Diagram 1 does not show the influence of the Reynolds number on the Sherwood number. However existing experimental data predict

$$Sh \propto Re^{0.75-0.88}$$

The higher the Schmidt number, the higher the Reynolds number exponent.

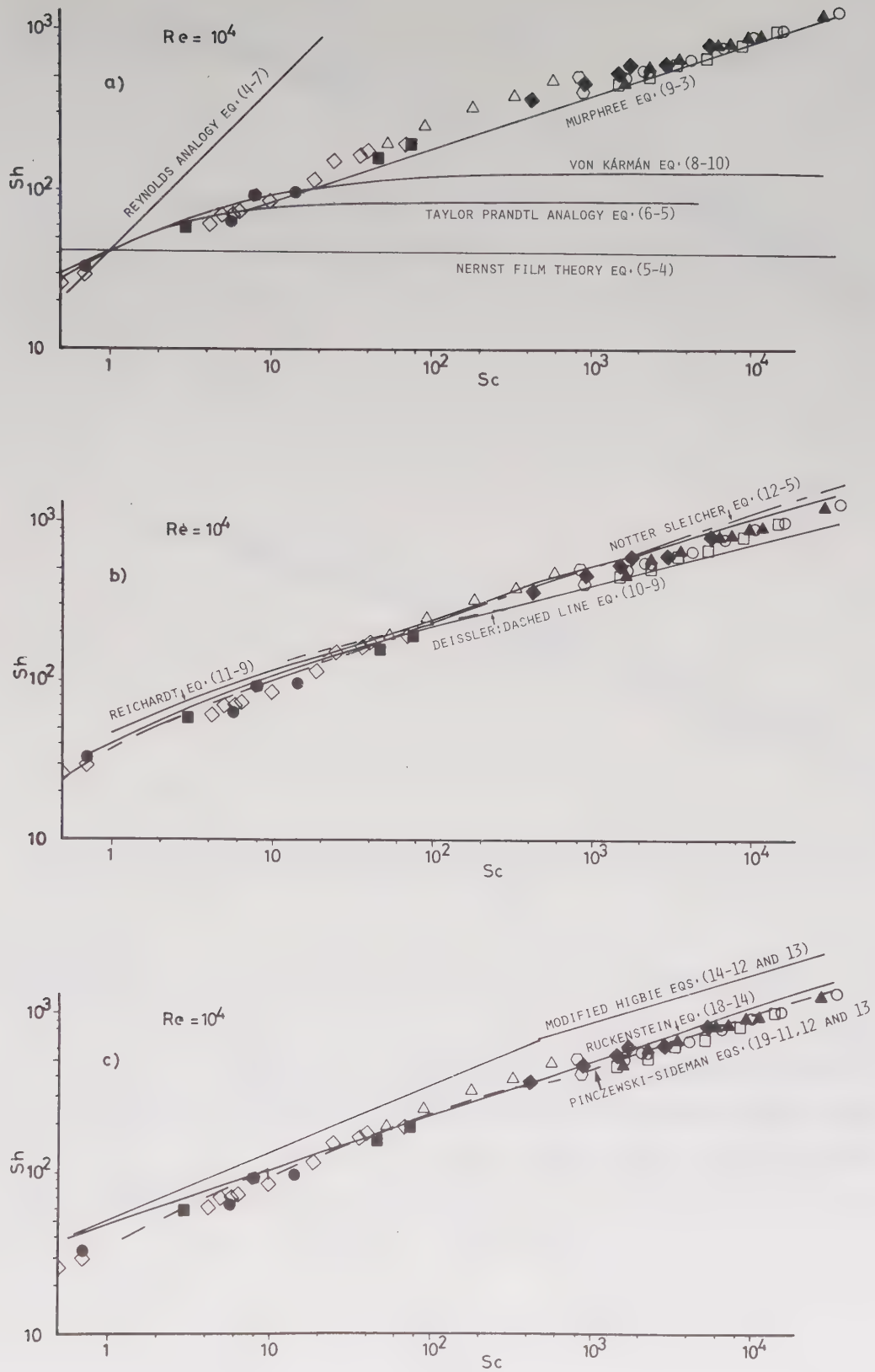


Diagram 1. Continued on next page

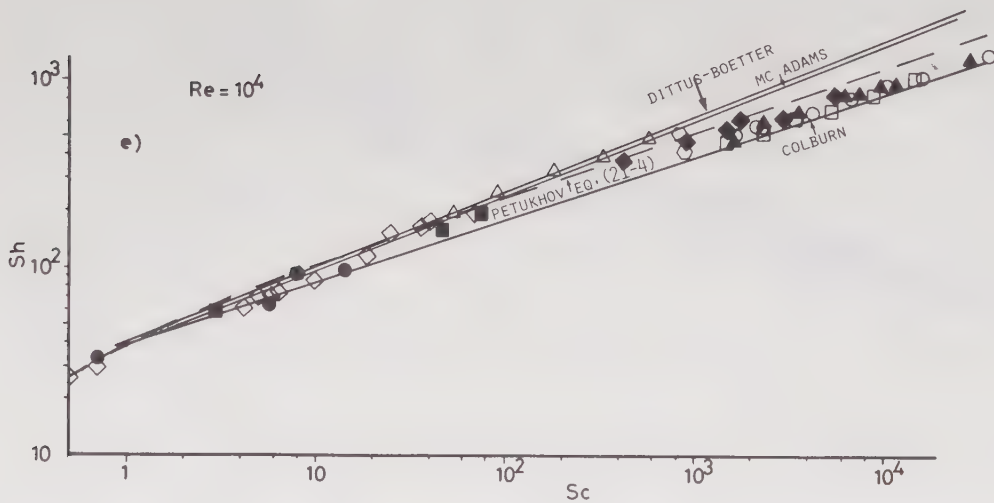
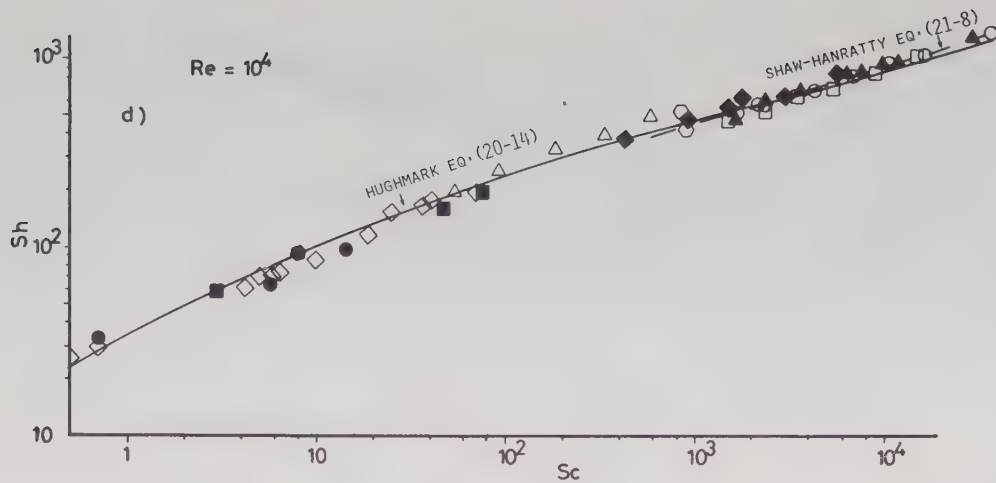


Diagram 1. Experimental mass and heat transfer data and some correlations.

- ◇ Friend and Metzner (1958, Table 2). The data might be too high because of no extrapolation to zero temperature difference.
- △ Friend and Metzner (1958). The data might be too high because of no extrapolation to zero temperature difference.
- Gowen and Smith (1968). The data might be too high because of no extrapolation to zero temperature difference.
- Malina and Sparrow (1964)
- ◆ Allen and Eckert (1964)
- ◆ Harriott and Hamilton (1965). Data at high Schmidt numbers are not reliable because of surface roughness.
- Meyerink and Friedlander (1962).
- Mizushina et al. (1971)
- ▲ Hubbard and Lightfoot (1966)
- Shaw and Hanratty (1977a)

ELEMENTARY MODELS

4. Reynolds Analogy

According to Reichardt (1961), Reynolds, at the end of the 19:th century was the first to realize a connection between mass and momentum transfer. He assumed that mass and momentum were exchanged between the bulk and the wall by eddies.

Per unit time and area n eddies, each with mass m , leave the bulk region and hit the wall. At constant density the same mass of fluid must go from the wall to the bulk region. The net flow of mass and momentum to the wall then will be

$$\tau_w = n \cdot m \cdot (u_b - u_w) \quad (4-1)$$

$$j_w = \frac{n \cdot m}{\rho} \cdot (c_b - c_w) \quad (4-2)$$

It is assumed no slip at the wall, i.e. $u_w = 0$

Equations (4-1 and 2) give

$$\frac{j_w \cdot \rho}{c_b - c_w} = \frac{\tau_w}{u_b} \quad (4-3)$$

Definitions of the fanning friction factor (f) at constant wall shear stress and the mass transfer coefficient (k) at nonpermeable walls give

$$\tau_w \equiv f \cdot \frac{1}{2} \cdot \rho \cdot u_b^2 \quad (4-4)$$

$$j_w \equiv k(c_b - c_w) \quad (4-5)$$

Equations (4-3, 4 and 5) give

$$k = f \cdot \frac{1}{2} \cdot u_b \quad (4-6)$$

As molecular diffusion is regarded as unimportant, the resulting mass transfer coefficient is independent of the mass diffusivity.

In dimensionless form Eq. (4-6) will be

$$Sh \equiv \frac{k \cdot d_h}{D} = \frac{1}{2} \cdot f \cdot Re \cdot Sc \quad (4-7)$$

Equation (4-7) is shown in Diagram 1a, and it is evident that the equation is valid only at $Sc = 1$. This is because the Reynolds analogy neglects the molecular diffusion, which is dominating near the wall. At $Sc = 1$ the mass and momentum molecular diffusivities are equal so the relationship between the mass and momentum transfer will be the same with or without regarding the molecular transport.

5. Nernst Film Theory

Nernst (1904) introduced the film theory, which assumes that the whole mass transfer resistance is in a thin film near the wall. Outside this film there is complete mixing so the mass concentration is c_b in the outer border of the film. Nernst assumed that the film was a stagnant layer, i.e. zero fluid velocity in the film, and that the film thickness was dependent on the bulk hydraulics. Later the film theory was modified by assuming laminar flow in the film, with the outer border velocity equal to the bulk velocity (u_b). With this assumption and assuming a thin enough film to let the shear stress and mass flux be constant in the film regardless of the wall curvature, Newton's law of viscosity and Fick's law give

$$\tau_w = \mu \cdot \frac{\partial u}{\partial y} = \mu \cdot \frac{u_b}{\delta} \quad y \leq \delta \quad (5-1)$$

$$j_w = D \cdot \frac{\partial c}{\partial y} = D \cdot \frac{c_b - c_w}{\delta} \quad y \leq \delta \quad (5-2)$$

δ = film thickness

zero slip velocity is assumed

The definitions of k and f according to Eqs. (4-4 and 5) together with Eqs. (5-1 and 2) give

$$k = \frac{1}{2} \cdot f \cdot u_b \cdot \frac{D}{\nu} \quad (5-3)$$

Thus the assumption of the whole mass transfer resistance in a laminar film gives that the mass transfer coefficient is directly proportional to the mass diffusivity.

In dimensionless form Eq. (5-3) will be

$$Sh = \frac{1}{2} \cdot f \cdot Re \quad (5-4)$$

Equation (5-4) is shown in Diagram 1a, and it is evident that the film theory gives as poor results as Reynolds analogy but in the opposite direction.

Later the film theory has been further modified, by assuming a different film thickness for the mass transfer than that for the momentum transfer, Yet the basic idea of the film theory is, that the whole mass transfer resistance is in a film where there is no turbulence.

6. Taylor-Prandtl Analogy

Prandtl (1910) combined Reynolds analogy and the film theory, so the mass and momentum transfer became dependent both on turbulent and molecular transport. He divided the cross sectional area in three regions

- 1) near the wall where the turbulent transport is neglected (the film theory)
- 2) one intermediate region where the molecular and the turbulent transports are in the same order of magnitude
- 3) the core region where the molecular transport is neglected (Reynolds analogy).

To be able to treat the problem mathematically Prandtl disregarded the intermediate region. He writes that by doing so he certainly makes a noticeable error. Prandtl's mathematical model thus consists of a laminar film (thickness δ) near the wall, and outside this a core region where Reynolds analogy is valid.

For the film Eqs. (5-1 and 2) give

$$\frac{j_w}{D(c_\delta - c_w)} = \frac{\tau_w}{\mu \cdot u_\delta} \quad (6-1)$$

In the core region Eqs. (4-1 and 2) give

$$\frac{j_w \cdot \rho}{c_b - c_\delta} = \frac{\tau_w}{u_b - u_\delta} \quad (6-2)$$

By combining Eqs. (6-1 and 2), c_{δ} is eliminated. Then the use of the definitions of f and k (Eqs. (4-4 and 5)) give

$$k = \frac{\frac{1}{2} \cdot f \cdot u_b}{1 + \frac{u_{\delta}}{u_b} (Sc-1)} \quad (6-3)$$

According to von Kármán (1939), Taylor obtained the same equation 1916, so it is now called the Taylor-Prandtl analogy. The quotient u_{δ}/u_b must be empirically determined, and according to existing experimental data Prandtl (1928) proposed

$$\frac{u_{\delta}}{u_b} = \text{const.} \cdot Re^{-\frac{1}{8}} \quad (6-4)$$

Equations (6-3 and 4) with the value of the constant equal to 1.5 give

$$Sh = \frac{\frac{1}{2} \cdot f \cdot Re \cdot Sc}{1 + 1.5 \cdot Re^{-\frac{1}{8}} \cdot (Sc-1)} \quad (6-5)$$

As seen in Diagram 1a the agreement between experimental data and Eq. (6-5) is good up to a Schmidt number of about five. At higher Schmidt numbers the Taylor-Prandtl analogy predicts too low mass transfer coefficients. This is because, the higher the Schmidt number the lower the mass diffusivity and consequently the more important is the turbulent transport of mass which is neglected in the wall region by Taylor-Prandtl analogy.

EDDY DIFFUSIVITY MODELS

7. The Law of the Wall

Near the wall the shear stress can be regarded as constant (τ_w). Then the fluid velocity (u) near the wall will depend on the wall shear stress (τ_w), the distance to the wall (y), the fluid density (ρ), the fluid kinematic viscosity (ν) and the turbulence near the wall. As earlier discussed in Chapter 2, the eddy frequency (n) near the wall is a function of $n=n(\tau_w, \nu, \rho, y)$. Then the unspecified "turbulence near the wall" ought to be a function of the same quantities and the function for the fluid velocity is

$$u = u(\tau_w, \nu, \rho, y)$$

Dimension analysis gives

$$\frac{u}{\sqrt{\tau_w/\rho}} = u\left(\frac{y \cdot \sqrt{\tau_w/\rho}}{\nu}\right) \quad (7-1)$$

As earlier mentioned $\sqrt{\tau_w/\rho}$ is named the friction velocity (u^*) and Eq. (7-1) can be rewritten as

$$u^+ = u(y^+) \quad (7-2)$$

This equation says that near the wall, the dimensionless fluid velocity u^+ is a function of only the dimensionless distance y^+ to the wall. This is called "The law of the wall" and was formulated in the 1930's.

The shear stress decreases with increasing distance from the wall

$$\tau = \tau_w \cdot \left(1 - \frac{y^+}{y_c^+}\right) \quad (7-3)$$

y_c = the centerline distance from the wall

Thus the shear stress is dependent not only on the distance from the wall but also on the duct size. For example Bogue and Metzner (1963) empirically obtained for the velocity profile in turbulent flow in circular ducts

$$u^+ = 5.57 \log y^+ + 5.57 + d\left(\frac{y}{y_c}, f\right) \quad y^+ > 30 \quad (7-4)$$

$$d\left(\frac{y}{y_c}, f\right) = 0.05\sqrt{\frac{2}{f}} \exp\left(-\frac{\left(\frac{y}{y_c} - 0.8\right)^2}{0.15}\right) \quad (7-5)$$

The correction term $d\left(\frac{y}{y_c}, f\right)$ can be seen as the deviation from the law of the wall. At $Re = 10^4$, $d_h = 10^{-2}$ m, $\nu = 10^{-6}$ m²/s the correction term is about 1% of the value of u^+ for $y^+ = 100$. Thus the correction term is usually negligible for $y^+ < 100$.

If the mechanism is the same for turbulent mass and momentum transfer, the function for the concentration profile near the wall is

$$c = c(\tau_w, \nu, \rho, y, D)$$

The concentration c is made dimensionless by dividing by $D \cdot \left(\frac{\partial c}{\partial y}\right)_w / u^*$. Hence as $D \cdot \left(\frac{\partial c}{\partial y}\right)_w = j_w$ at nonpermeable walls

$$c^+ = \frac{c \cdot u^*}{j_w} \quad (\text{definition, } v_y = 0) \quad (7-6)$$

Dimensional analysis then gives

$$c^+ = c(y^+, Sc) \quad (7-7)$$

At fully developed turbulent duct flow with nonpermeable walls and constant physical properties, mass and momentum are transferred radially only by molecular and turbulent transport.

$$\tau = (\mu + \rho \cdot \epsilon_m) \cdot \frac{du}{dy} \quad (7-8)$$

$$j = (D + \epsilon_q) \cdot \frac{dc}{dy} \quad (7-9)$$

ϵ_m and ϵ_q are called the eddy momentum and mass diffusivities respectively and they are defined by Eqs. (7-8 and 9). Those models which are based on Eqs. (7-8 and 9) are called eddy diffusivity models.

Near the wall the wall curvature is unimportant, so there rectangular coordinates can be used even for circular duct flow.

In dimensionless quantities Eqs. (7-8 and 9) will be

$$\frac{\tau}{\tau_w} = \left(1 + \frac{\epsilon_m}{\nu}\right) \cdot \frac{du^+}{dy^+} \quad (7-10)$$

$$\frac{j}{j_w} = \left(\frac{1}{Sc} + \frac{\epsilon_q}{\nu}\right) \cdot \frac{dc^+}{dy^+} \quad (7-11)$$

Reynolds analogy, Nernst film theory and Taylor-Prandtl analogy are obtained from Eqs. (7-10 and 11) at the following conditions

Reynolds analogy

$$\frac{\tau}{\tau_w} \cdot \frac{j_w}{j} \cdot \frac{D + \epsilon_q}{\nu + \epsilon_m} = 1 \quad \text{which gives } c^+ = u^+ + \text{const.}$$

Nernst film theory

$$\left. \begin{array}{l} \frac{\tau}{\tau_w} = \frac{j}{j_w} \\ y^+ < \delta^+ \quad \epsilon_m = \epsilon_q = 0 \\ y^+ > \delta^+ \quad u^+ = u_b^+, \quad c^+ = c_b^+ \end{array} \right\} \begin{array}{l} \text{which give } u^+ = \frac{1}{Sc} \cdot (c^+ - c_w^+) \\ \text{for } y^+ < \delta^+ \end{array}$$

Taylor-Prandtl analogy

$$\left. \begin{array}{l} \frac{\tau}{\tau_w} = \frac{j}{j_w}, \quad \epsilon_m = \epsilon_q \\ y^+ < \delta^+ \quad \epsilon_m = 0 \\ y^+ > \delta^+ \quad \frac{\epsilon_m}{\nu} \gg 1, \quad \frac{\epsilon_q}{D} \gg 1 \end{array} \right\} \begin{array}{l} \text{which give Nernst film theory for} \\ y^+ < \delta^+ \text{ and Reynolds analogy for} \\ y^+ > \delta^+ \end{array}$$

If the whole concentration boundary layer is within the region where the shear stress can be assumed to be constant, Eq. (7-7) gives

$$c_b^+ - c_w^+ = c_1(Sc) \quad (7-12)$$

As $c_b^+ - c_w^+ = u^*/k$, Eq. (7-12) gives

$$k = u^* \cdot c_2(Sc) \quad (7-13)$$

Thus for constant Schmidt number, the mass transfer coefficient is directly proportional to the friction velocity or $\sqrt{\frac{f}{2}} \cdot u_b$.

8. von Kármán's analysis

von Kármán (1939) completed Prandtl's model by dividing the duct cross sectional area in three regions. One nearest the wall with molecular transport only, one intermediate region with both molecular and turbulent transport and the core region with turbulent transport only. Experimental velocity profile data for $y^+ > 10$ are available. According to these, the law of the wall and Reynolds analogy von Kármán assumes

$$u^+ = y^+ \quad y^+ < 5 \quad (8-1)$$

$$u^+ = 5 \left(1 + \ln \frac{y^+}{5} \right) \quad 5 < y^+ < 30 \quad (8-2)$$

$$u^+ = c^+ + \text{const. (Reynolds analogy)} \quad y^+ > 30 \quad (8-3)$$

For the two regions near the wall, it is further assumed

$$\frac{\tau}{\tau_w} = \frac{j}{w} = 1, \quad \epsilon_m = \epsilon_q \quad y^+ < 30 \quad (8-4)$$

Equations (7-10) and (8-1, 2 and 4) give the expression for the eddy diffusivities

$$\left. \begin{aligned} \frac{\epsilon_m}{\nu} &= 0 & y^+ < 5 \\ \frac{\epsilon_m}{\nu} &= \frac{y^+}{5} - 1 & 5 < y^+ < 30 \end{aligned} \right\} \quad (8-5)$$

Hence Eqs. (7-11) and (8-4 and 5) give

$$\int_0^{y^+=30} dc^+ = Sc \int_0^5 dy^+ + \int_5^{30} \frac{dy^+}{\frac{1}{Sc} + \frac{y^+}{5} - 1} \quad (8-6)$$

$$c_{y^+=30}^+ - c_w^+ = 5 \cdot Sc + 5 \cdot \ln (5 \cdot Sc + 1) \quad (8-7)$$

Equations (8-7, 2 and 3) with $c^+ = c_b^+$ at $u^+ = u_b^+$ then give

$$c_b^+ - c_w^+ = u_b^+ - 5 \cdot (1 + \ln 6) + 5 \cdot Sc + 5 \cdot \ln (5 \cdot Sc + 1) \quad (8-8)$$

Equation (8-8) with the definition of c^+ , j_w and τ_w according to Eqs. (7-6) and (4-5 and 4) respectively give

$$k = \frac{\frac{f}{2} \cdot u_b}{1 + 5 \sqrt{\frac{f}{2}} \cdot (Sc - 1 + \ln(1 + \frac{5}{6} \cdot (Sc - 1)))} \quad (8-9)$$

In dimensionless form Eq. (8-9) will be

$$Sh = \frac{\frac{1}{2} \cdot f \cdot Re \cdot Sc}{1 + 5 \sqrt{\frac{f}{2}} [Sc - 1 + \ln(1 + \frac{5}{6} (Sc - 1))]} \quad (8-10)$$

Equation (8-10) is shown in diagram 1a and there it can be seen that this equation correlates experimental data better than Taylor-Prandtl analogy does, but Eq. (8-10) is not valid for Schmidt numbers above 10. This is because von Kármán, like Taylor and Prandtl assumes that there is a region near the wall where no turbulent mass transfer exists. If so, at high Schmidt numbers, the whole mass transfer resistance should be within this wall region. Then the mass transfer coefficient should be directly proportional to the mass molecular diffusivity and thus the Sherwood number independent on the Schmidt number. Experimental data have shown that this is not the case. von Kármán himself emphasized that his result was not reliable at high Schmidt numbers, but the model could be improved when experimental data near the wall were available.

9. Murphree's analysis

Before von Kármán's paper, Murphree (1932) pointed out that the turbulent mass transfer was of importance all the way to the wall. He assumed for the eddy diffusivities

$$\varepsilon_q = \varepsilon_m$$

$$\varepsilon_m = (\varepsilon_m)_b \cdot \left(\frac{y}{\delta}\right)^a \quad y < \delta \quad (9-1)$$

$$\epsilon_m = (\epsilon_m)_b \quad y > \delta \quad (9-2)$$

$(\epsilon_m)_b$ is a constant value in the bulk region. Thus he divided the cross sectional area into two regions. In both regions, both molecular and turbulent transfer were considered. In order to simplify the calculations Murphree assumed that the exponent "a" in Eq. (9-1) was an integer. On physical considerations he assumed that "a" must be three or more. He chose $a = 3$ only because this value of "a" seemed to give better mass transfer predictions than higher values.

For $y < \delta$ it was assumed that the shear stress was constant $(\tau_w + \tau_\delta)/2$. By Eqs. (9-1 and 2) and

$$\tau = (\mu + \rho \epsilon_m) \cdot \frac{du}{dy} \quad (7-8)$$

it is then possible to calculate the velocity profile as a function of τ_w , δ and $\frac{(\epsilon_m)_b}{v}$. From experimental velocity profile data Murphree then calculated δ and $\frac{(\epsilon_m)_b}{v}$ by the trial and error method.

Murphree then assumed that the whole mass transfer resistance was in the region $y < \delta$, i.e. $c_\delta = c_b$, and within this region the mass flux was constant $j_w \cdot \frac{y_c}{y_c - \delta/2}$. As $\epsilon_q = \epsilon_m$, Eq. (7-9) gives the correlation for the mass transfer coefficient. This is tabulated by Murphree, and for $Sc > 10$ and $Re > 10^4$ the result correlates within 2% with Eq. (9-3)

$$Sh = 0.0175 \cdot (1 - 11.1 \cdot Re^{-0.57}) \cdot Re^{0.836} \cdot Sc^{\frac{1}{3}} \quad \begin{cases} Sc > 10 \\ Re > 10^4 \end{cases} \quad (9-3)$$

Equation (9-3) is shown in Diagram 1a, and the agreement with experimental data is fairly good at high Schmidt numbers. This is because Murphree considered the turbulent mass transfer near the wall.

10. Deissler's analysis

Deissler (1961), in the beginning of the 1950's divided the cross sectional area into two regions. In the wall region ($u^+ < \delta^+$) both molecular and turbulent mass and momentum transfer were considered, like Murphree's analysis, and in the core region ($y^+ > \delta^+$) Reynolds

analogy was assumed to be valid. For the wall region it was assumed

$$\epsilon_m = \epsilon(u, y, \nu) \quad y^+ < \delta^+ \quad (10-1)$$

which is consistent with the law of the wall

Dimensional analysis gives

$$\epsilon_m = a^2 \cdot u \cdot y \cdot \epsilon\left(\frac{a^2 u y}{\nu}\right) \quad (10-2)$$

where "a" is a constant

Deissler assumed

$$\begin{aligned} \epsilon\left(\frac{a^2 u y}{\nu}\right) &\rightarrow 0 \quad \text{as } y \rightarrow 0 \\ \epsilon\left(\frac{a^2 u y}{\nu}\right) &\rightarrow 1 \quad \text{as } y \text{ increases} \end{aligned}$$

and chose a simple function which gave the above limits.

$$\epsilon\left(\frac{a^2 u y}{\nu}\right) = 1 - \exp\left(-\frac{a^2 u y}{\nu}\right) \quad (10-3)$$

The constant "a" was chosen to fit experimental data.

Equations (10-2 and 3) and (7-10) give

$$\frac{\tau}{\tau_w} = (1 + a^2 u^+ y^+ \cdot [1 - \exp(-a^2 u^+ y^+)]) \frac{du^+}{dy^+} \quad y^+ < \delta^+ \quad (10-4)$$

Further it is assumed that $\frac{\tau}{\tau_w} = 1$ for $y^+ < \delta^+$. Equation (10-4) correlated with experimental velocity profile data then give

$$\delta^+ = 26$$

$$a = 0.124$$

From

$$\frac{j}{j_w} = \left(\frac{1}{Sc} + \frac{\epsilon_q}{\nu}\right) \frac{dc^+}{dy^+} \quad (7-11)$$

$$\frac{j}{j_w} = 1, \quad \epsilon_q = \epsilon_m \quad y^+ < \delta^+.$$

$$\text{Reynolds analogy} \quad y^+ > \delta^+$$

it is possible to calculate the concentration profile and the mass transfer coefficient. It is not possible to do this analytically and the result obtained by a numerical method is shown in Diagram 1b. There it is evident that Deissler's analysis gives good estimations of the mass transfer coefficient up to a Schmidt number of about 100. The reason for the bad estimations at higher Schmidt numbers is, that the velocity profile is insensitive to the eddy diffusivity very near the wall, because there the momentum molecular diffusivity is quite dominant. Thus it is impossible to obtain reliable eddy diffusivity data near the wall from velocity profile data, as Deissler and others have tried to do. Though there are better mass transfer correlations today than those which Deissler's analysis gives, many authors still use Deissler's correlation when comparing their own experimental data at high Schmidt numbers. Therefore Deissler's result at high Schmidt numbers is presented here.

At high Schmidt numbers the whole concentration boundary layer is very near the wall where

$$\left. \begin{aligned} u^+ &= y^+ \\ \exp(-a^2 y^{+2}) &\approx 1 - a^2 y^{+2} \\ \frac{j}{j_w} &= 1 \end{aligned} \right\} \quad (10-5)$$

If $\epsilon_q = \epsilon_m$ Eqs. (7-11) and (10-2 and 3) give

$$dc^+ = \frac{dy^+}{\frac{1}{Sc} + a^4 y^{+4}} \quad (10-6)$$

The boundary conditions are

$$\begin{array}{lll} \text{B.C. 1} & \text{at } y^+ = 0 & c^+ = c_w^+ \\ \text{B.C. 2} & \text{at } y^+ = \infty & c^+ = c_b^+ \end{array}$$

B.C. 2 is chosen for the simplicity in evaluation of the integral

$\int_0^{\infty} \frac{dy^+}{\frac{1}{Sc} + a y^{+4}}$. At high Schmidt numbers Eq. (10-6) with B.C. 1 and 2 gives $c^+ \approx c_b^+$ at such low values of y^+ that Eq. (10-5) is still valid. Thus B.C. 2 is acceptable at high Schmidt numbers.

Equation (10-6) with B.C. 1 and 2 gives

$$c_b^+ - c_w^+ = \frac{\pi}{2\sqrt{2} \cdot a} Sc^{\frac{3}{4}} \quad (10-7)$$

Equation (10-7) together with the definition of c^+ (Eq. (7-6)), j_w (Eq. (4-5)) and u^* ($\sqrt{\frac{\tau}{2}} \cdot u_b$) and $a = 0.124$ give

$$k = 0.112 \sqrt{\frac{\tau}{2}} \cdot u_b \cdot Sc^{-\frac{3}{4}} \quad (10-8)$$

In dimensionless form eq. (10-8) will be

$$Sh = 0.122 \sqrt{\frac{\tau}{2}} \cdot Re \cdot Sc^{\frac{1}{4}} \quad (10-9)$$

Equations (10-9) is presented as a dashed line in Diagram 1b and it is evident that this equation agrees with Deissler's exact solution at $Sc > 100$.

11. Reichardt's analysis

Reichardt 1951 published a paper (Reichardt (1961)) where he derived an expression for the mass transfer coefficient, which was independent of assumed velocity profile and even varying physical properties. The requirements are

- nonpermeable walls
- smooth walls
- the wall concentration changes so slowly that there is no axial diffusion.

Equations (7-10 and 11) and the definitions

$$v_e \equiv v + \epsilon_m$$

$$D_e \equiv D + \epsilon_q$$

$$Sc_e \equiv \frac{v_e}{D_e}$$

give

$$\frac{\tau}{\tau_w} = \frac{v_e}{v} \frac{du^+}{dy^+} \quad (11-1)$$

$$\frac{j}{j_w} = \frac{D_e}{v} \frac{dc^+}{dy^+} \quad (11-2)$$

Equation (11-2) is rearranged, both sides multiplied by $\frac{\epsilon_q}{\epsilon_m}$ and integrated over y^+

$$\int_0^{y_c^+} \frac{\epsilon_q}{\epsilon_m} \frac{dc^+}{dy^+} dy^+ = \int_0^{y_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot \frac{v}{D_e} \cdot \frac{j}{j_w} dy^+ \quad (11-3)$$

The left side of Eq. (11-3) is evolved

$$\int_0^{y_c^+} \frac{\epsilon_q}{\epsilon_m} \frac{dc^+}{dy^+} dy^+ = \int_{c_w^+}^{c_c^+} \frac{\epsilon_q}{\epsilon_m} dc^+ = (c_c^+ - c_w^+) \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle \quad (11-4)$$

Equation (11-4) defines $\left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle$ which is an average value of $\frac{\epsilon_q}{\epsilon_m}$ in the cross section.

"a" is defined as

$$\frac{j}{j_w} \equiv (1 + a) \cdot \frac{\tau}{\tau_w} \quad (11-5)$$

The right side of Eq. (11-3) and Eqs. (11-1 and 5) give

$$\begin{aligned} \int_0^{y_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot \frac{v}{D_e} \cdot \frac{j}{j_w} dy^+ &= \int_0^{u_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot Sc_e \cdot (1 + a) du^+ = \int_0^{u_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot Sc_e du^+ + \\ &+ \int_0^{u_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot Sc_e \cdot a du^+ = \int_0^{u_c^+} du^+ + \int_0^{u_c^+} \left(\frac{\epsilon_q}{\epsilon_m} \cdot Sc_e - 1 \right) du^+ + \xi \cdot u_c^+ \end{aligned} \quad (11-6)$$

$$\xi \equiv \frac{1}{u_c^+} \cdot \int_0^{u_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot Sc_e \cdot a \, du^+$$

Equations (11-3, 4 and 6) give

$$(c_c^+ - c_w^+) \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle = \xi \cdot u_c^+ + \int_0^{u_c^+} \left(\frac{\epsilon_q}{\epsilon_m} \cdot Sc_e - 1 \right) du^+ + u_c^+ \quad (11-7)$$

Equation (11-7) with definitions of c^+ (Eq. (7-6)), j_w (Eq. (4-5)) and $u^* (\sqrt{\frac{f}{2}} \cdot u_b)$ gives

$$k = \frac{\frac{f}{2} \cdot u_b \cdot \frac{u_{\text{quot}}}{c_{\text{quot}}} \cdot \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle}{1 + \xi + \frac{1}{u_c^+} \int_0^{u_c^+} \left(\frac{\epsilon_q}{\epsilon_m} \cdot Sc_e - 1 \right) du^+} \quad (11-8)$$

$$u_{\text{quot}} = \frac{u_b}{u_c}$$

$$c_{\text{quot}} = \frac{c_b - c_w}{c_c - c_w}$$

If $\epsilon_q = \epsilon_m$, the integrand in the denominator of Eq. (11-8) will be $(Sc - 1) \cdot \frac{D}{D + \epsilon_q}$. Therefore Reichardt put $(Sc - 1)$ outside the integral.

However Eq. (11-8) is valid also for varying physical properties, so the Schmidt number need not be a constant. Instead the factor $(Sc_f - 1)$ is put outside the integral. Sc_f is an average value of Sc near the wall, as the integrand decreases with increasing distance from the wall.

Furthermore b is defined as

$$b \equiv \int_0^{u_c^+} \frac{\frac{\epsilon_q}{\epsilon_m} \cdot Sc_e - 1}{Sc_f - 1} du^+$$

b is a function of the Schmidt and Reynolds number; $b = b(Re, Sc)$.

Equation (11-8) now will be

$$k = \frac{\frac{f}{2} \cdot u_b \cdot \frac{u_{\text{quot}}}{c_{\text{quot}}} \cdot \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle}{1 + \xi + u_{\text{quot}} \cdot \sqrt{\frac{f}{2}} \cdot (Sc_f - 1) \cdot b(Re, Sc)} \quad (11-9)$$

From Eq. (11-9) all eddy-diffusivity models (those which are based on Eqs. (7-8 and 9)) can be derived. Even for the limit $\epsilon_q = \epsilon_m = 0$ (laminar flow) Eq. (11-9) is applicable at axisymmetrical flow.

To be able to use Eq. (11-9) some quantities must be empirically determined.

$$\left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle$$

From experimental data Reichardt assumes that

$$\frac{\epsilon_q}{\epsilon_m} \rightarrow 1 \text{ as } y \rightarrow 0$$

$$\frac{\epsilon_q}{\epsilon_m} = 2 \text{ at free turbulence (in the core region)}$$

At high Schmidt numbers the whole concentration boundary layer is near the wall so

$$\left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle \rightarrow 1 \text{ as } Sc \rightarrow \infty$$

$$\text{At } Sc = 1 \text{ Reichardt assumes } \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle \approx 1.3$$

ξ_{--}

$$\xi = \frac{1}{u_c^+} \cdot \int_0^{u_c^+} \frac{\epsilon_q}{\epsilon_m} \cdot Sc_e \cdot a \, du^+$$

$$\frac{j}{j_w} = (1+a) \cdot \frac{\tau}{\tau_w}$$

From a mass balance, iterative calculations and experimental data the value of "a" is obtained. It is very near zero near the wall where Sc_e can be high. At a longer distance from the wall $Sc_e \approx \frac{\epsilon_m}{\epsilon_q}$. Hence

$$\xi \approx \frac{1}{u_c^+} \cdot \int_0^{u_c^+} a \, du^+ \quad (11-10)$$

For flow in circular ducts Reichardt obtained the result shown in Table 11-1.

Re Sc	1	10	100
10^4	0.07	0.04	0.03
10^5	0.04	0.03	0.02

Table 11-1. $\xi = \xi (Re, Sc)$ at flow in circular ducts according to Reichardt

Thus ξ is small at high Schmidt numbers.

b (Re, Sc)

$$b = \int_0^{u_c^+} \frac{\frac{\epsilon_q}{\epsilon_m} \cdot Sc_e^{-1}}{Sc_f^{-1}} du^+$$

It is only possible to determine b from experimental data. However at $Re \cdot Sc > 2500$ the integrand is very small besides very near the wall, and b then becomes independent of the Reynolds number

$$b = f (Sc) \quad Re \cdot Sc > 2500$$

At $\epsilon_q = \epsilon_m$ and constant $Sc > 1$, Reichardt approximately obtains from experimental data

$$b = 10 \cdot Sc^{-0.3} \quad Sc > 1 \quad (11-11)$$

u_{quot} and c_{quot}

u_{quot} and c_{quot} are determined from calculated velocity and concentration profiles respectively. The figures in Table 11-2 are calculated from Reichardt (1961, Figs. 7 and 9)

Re Sc	1	10	10^2	∞
10^4	0.95	0.83	0.80	0.78
10^5	1.02	0.90	0.86	0.85
10^6	1.00	0.92	0.88	0.87

Table 11-2. $u_{quot}/c_{quot} = f (Re, Sc)$ for flow in circular ducts according to Reichardt

At $Sc = \infty$ $c_{quot} = 1$ so u_{quot} is obtained from the last column in Table 11-2.

The result of Eq. (11-9) and Reichardt's choice of values of the quantities which must be empirically determined, is shown in Diagram 1b. Reichardt's result overestimates the mass transfer coefficient at $Sc < 50$. This is probably because of his high value of $\left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle$ at low Schmidt numbers.

At high Schmidt numbers Eqs. (11-9 and 11) give

$$k = 0.1 \cdot \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{-0.7} \quad (11-12)$$

In dimensionless form Eq. (11-12) will be

$$Sh = 0.1 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{0.3} \quad (11-13)$$

At $Re = 10^4$ Eq. (11-12) gives 4% and 1% higher values than Eq. (11-9) at $Sc = 100$ and $Sc = 10^3$ respectively.

12. Conclusion of Eddy Diffusivity Models

There are many other works based on the eddy diffusivity model for calculating the mass transfer coefficient. Some of them divide the cross sectional area in up to six sections, each with expressions for the eddy momentum and mass diffusivities. However these models do not further clarify the transport mechanism by turbulent flow, and the eddy diffusivities must still be empirically determined. Hubbard and Lightfoot (1966) and Webb (1971) critically examine some eddy diffusivity models, and there references can be found to some works not considered here. The work of Reichardt is still the most universal of the eddy diffusivity models, but to be reliable the model needs reliable eddy diffusivity data. Experimental investigations indicates that ϵ_q is proportional to y^+ ³ near the wall and to y^+ ² far away from the wall. Notter and Sleicher (1971) arbitrarily chose such a function

$$\frac{\epsilon_q}{\nu} = \frac{a \cdot y^{+3}}{\left[1 + \left(\frac{a}{0.011} \right)^2 \cdot y^{+2} \right]^{\frac{1}{2}}} \quad y^+ < 45 \quad (12-1)$$

$a = \text{constant}$

At $y^+ > 45$ the expression for ϵ_q is not important for $Sc > 1$, as there the bulk concentration is reached. Notter and Sleicher used an empirically obtained velocity profile and a computer to solve the equation of continuity for a solute, assuming no axial diffusion. The best agreement between the calculations and experimental data occurred for $a = 0.9 \cdot 10^{-3}$. At high Schmidt numbers the denominator in Eq. (12-1) is equal to one in the concentration boundary layer and then Eq. (7-11), $\frac{j}{j_w} = 1$ and $a = 0.9 \cdot 10^{-3}$ give

$$dc^+ = \frac{dy^+}{\frac{1}{Sc} + 0.9 \cdot 10^{-3} y^{+3}} \quad (12-2)$$

Integration of Eq. (12-2) gives

$$k = 0.080 \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{\frac{2}{3}} \quad (12-3)$$

Notter and Sleicher claim that their result at $Sc > 100$ correlates within 4% with

$$Sh = 0.0149 \cdot Re^{0.88} \cdot Sc^{\frac{1}{3}} \quad (12-4)$$

With Blasius expression for the fanning friction factor $f = 0.079 \cdot Re^{-0.25}$ the difference between Eqs. (12-3) and (12-4) is within 2% for $5 \cdot 10^3 < Re < 10^7$.

In later works, Notter and Sleicher (1972) and Sleicher and Rouse (1975), in the same way found that the following equation correlates experimental data within 10%.

$$Sh = 5 + 0.015 \cdot Re^a \cdot Sc^b \quad \left\{ \begin{array}{l} 0.1 < Sc < 10^4 \\ 10^4 < Re < 10^6 \end{array} \right. \quad (12-5)$$

$$a = 0.88 - \frac{0.24}{Sc+4}$$

$$b = \frac{1}{3} + 0.5 \exp(-0.6 Sc)$$

Equation (12-5) is shown in Diagram 1b, and it is evident that this equation at high Schmidt numbers predicts higher mass transfer coefficients than the experimental data included. This is because Notter

and Sleicher based their correlations on experimental data other than those presented here in Diagram 1.

Notter and Sleicher, among others, first use experimental data to obtain expressions for the eddy diffusivities. Then they use these expressions to obtain mass and heat transfer correlations. This way is quite unnecessary, because the same mass and heat transfer correlations are more easily obtained directly from experimental data. Furthermore, the eddy diffusivities are just definitions and not real quantities, so they are not observable or measurable. The only use of empirically determined eddy diffusivities is perhaps in calculating the average concentration profile.

To conclude, the eddy diffusivity models give a good understanding of the momentum, mass and heat transfer in turbulent flow, but it is of no use to produce correlations for the eddy diffusivities from experimental data.

PENETRATION MODELS

13. The Penetration Theory

The penetration theory can be seen as a development of Reynolds analogy. Like Reynolds analogy the penetration theory assumes that mass and momentum are transferred by fluid elements or whirls. The difference is the treatment of the mass and momentum transfer between the whirl and the wall or surrounding fluid. Reynolds analogy assumes that this occurs instantaneously, while the penetration theory assumes that this occurs by molecular diffusion. Originally the penetration theory did not consider the momentum transfer, but in later models momentum was also supposed to be transferred by molecular diffusion.

14. Higbie's and Danckwerts analyses

The penetration theory was originally proposed by Higbie (1935) for application on gas-liquid mass transfer. He assumed that liquid elements left the liquid bulk for the liquid-gas interface and that the contact time, liquid element-gas, was too short to produce stationary conditions. Then the two-dimensional equation of continuity for the solute in a binary mixture with constant physical properties and without internal chemical reaction will be

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v_y \frac{\partial c}{\partial y} = D \left(\frac{\partial^2 c}{\partial x^2} + \frac{\partial^2 c}{\partial y^2} \right) \quad (14-1)$$

If $v_y = 0$ and the concentration c is independent of x , Eq. (14-1) will be

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2} \quad (14-2)$$

Higbie assumes that the liquid element is thick enough, so the concentration in its region far away from the interface (inner region) is unchanged throughout contact time.

Hence the initial and boundary conditions of Eq. (14-2) will be

I.C.	$c = c_b$	at	$t = 0$
B.C. 1	$c = c_w$	at	$y = 0$
B.C. 2	$c = c_b$	at	$y = \infty$

The solution of Eq. (14-2) with the conditions above is

$$\frac{c(y,t) - c_w}{c_b - c_w} = \text{erf} \frac{y}{2\sqrt{Dt}} \quad (14-3)$$

$$\text{erf}(\psi) \equiv \frac{2}{\sqrt{\pi}} \cdot \int_0^{\psi} \exp(-\psi^2) d\psi$$

The mass transfer coefficient is obtained from

$$k = \frac{D \left. \frac{\partial c}{\partial y} \right|_w}{c_b - c_w} \quad (14-4)$$

Equations (14-3 and 4) give

$$k = \sqrt{\frac{D}{\pi t}} \quad (14-5)$$

Thus the mass transfer coefficient varies with the contact time. At contact time θ the time average value of the mass transfer coefficient will be

$$\bar{k} = \frac{1}{\theta} \cdot \int_0^{\theta} k dt \quad (14-6)$$

$$\bar{k} = 2\sqrt{\frac{D}{\pi\theta}} \quad (14-7)$$

Higbie's analysis is applicable for mass transfer in turbulent duct flow. Here it is assumed that fluid elements are periodically exchanged between the bulk and the wall. The contact time (θ) is assumed constant and its value can be estimated as the inverse of the eddy frequency near the wall described in Chapter 2. The result for mass transfer was

$$\bar{n}_w^+ \approx 0.005 \cdot Sc^{-0.172} \quad 1 < Sc < 500 \quad (2-2)$$

$$\bar{n}_w^+ \approx 0.022 \cdot Sc^{-0.41} \quad Sc > 500 \quad (2-1)$$

If $\theta = (\bar{n}_w^+)^{-1}$, Eqs. (2-1 and 2) give

$$\theta = \frac{v \cdot Sc^{0.172}}{0.005 \cdot u_*^2} \quad 1 < Sc < 500 \quad (14-8)$$

$$\theta = \frac{v \cdot Sc^{0.41}}{0.022 \cdot u^{*2}} \quad Sc > 500 \quad (14-9)$$

Equations (14-7, 8 and 9) give

$$\bar{k} = 0.080 \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{-0.59} \quad 1 < Sc < 500 \quad (14-10)$$

$$\bar{k} = 0.167 \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{-0.70} \quad Sc > 500 \quad (14-11)$$

In dimensionless form Eqs. (14-10 and 11) will be

$$Sh = 0.080 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{0.41} \quad 1 < Sc < 500 \quad (14-12)$$

$$Sh = 0.167 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{0.30} \quad Sc > 500 \quad (14-13)$$

Equations (14-12 and 13) are shown in Diagram 1c. The slope of the line, i.e. the Schmidt number exponent, agrees well with experimental data, but the equations predict too high mass transfer coefficients. The main reasons for this are

- the model assumes that the whirls come from the bulk region ($c = c_b$) and as mentioned in chapter 2 they actually come from the generation region $5 < y^+ < 70$. At high Schmidt numbers the concentration at $y^+ = 5$ is equal to c_b , but at lower Schmidt numbers the deviation from $c = c_b$ in the generation region increases with decreasing Schmidt numbers.
- the model assumes that all whirls from the bulk region reach the wall. As most of the whirls actually do not reach the wall, an overprediction of the mass transfer coefficient is made. However this is partly compensated for by the empirical correlation of the whirl frequency near the wall, but obviously the model is too simple to be applicable. At $Sc > 10^3$ the model predicts twice as high mass transfer coefficients compared to experimental data.

Danckwerts (1951) modified Higbie's model. He assumed that the contact time varied but the probability for fluid element exchange between bulk region and interface was independent of the "age" of the element at the interface. Further he assumed that the interface fraction renewed per unit time was constant (denoted s). Then it can be shown that

$$\bar{k} = \sqrt{D \cdot s} \quad (14-14)$$

The reasonable assumption $s = \bar{n}_w$ and Eq. (14-14) result in an 11% lower mass transfer coefficient than predicted by Eqs. (14-10 and 11).

15. The Penetration Frequency

Hanratty (1956) and Einstein and Li (1956) calculated the wall eddy frequency according to the penetration theory. The wall region is assumed thin enough, so rectangular coordinates can be used regardless of the wall curvature. The two-dimensional equation of motion will then be

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v_y \frac{\partial u}{\partial y} = - \frac{1}{\rho} \frac{\partial P}{\partial x} + g + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (15-1)$$

The penetration theory assumes u independent of x and the transverse velocity v_y equal to zero. Further, if the pressure gradient and the gravity are disregarded, Eq. (15-1) will be

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial y^2} \quad (15-2)$$

At time $t = 0$ a fluid element from a region with $u = u_0$ penetrates to the wall. The fluid element is assumed thick enough, so the fluid velocity in its inner region is not affected during the wall contact time. The time for moving to and from the wall is negligible compared to the wall contact time. The initial and boundary conditions will then be

I.C.	$u = u_0$	at	$t = 0$
B.C. 1	$u = 0$	at	$y = 0$
B.C. 2	$u = u_0$	at	$y = \infty$

Equation (15-2) and its initial and boundary conditions are exactly the same as Eq. (14-2) and its initial and boundary conditions. Thus the solution of Eq. (15-2) will be

$$u(y, t) = u_0 \cdot \text{erf} \frac{y}{2\sqrt{\nu t}} \quad (15-3)$$

The wall shear stress is

$$\tau_w = \mu \left. \frac{\partial u}{\partial y} \right|_w \quad (15-4)$$

The wall contact time (θ) is assumed constant

$$\overline{\tau}_w = \frac{1}{\theta} \int_0^{\theta} \tau_w dt = 2 \cdot u_o \cdot \rho \cdot \sqrt{\frac{\nu}{\pi \theta}} \quad (15-5)$$

As $\frac{\overline{\tau}_w}{\rho} = u^{*2}$, Eq. (15-5) gives

$$\theta = \frac{4 \cdot u_o^2 \cdot \nu}{\pi \cdot u^{*4}} \quad (15-6)$$

The dimensionless wall penetration frequency is

$$\overline{n}_w^+ = \frac{\nu}{\theta \cdot u^{*2}} \quad (15-7)$$

Equations (15-6 and 7) give

$$\overline{n}_w^+ = \frac{\pi}{4} \cdot \frac{1}{u_o^+{}^2} \quad (15-8)$$

Einstein and Li assumed that the fluid elements came from the bulk region, which gives $u_o = u_b$. Hanratty assumed that they came from a region near the wall, at about $y^+ = 30$. This well agrees with previously mentioned experimental investigations, which have also shown that $u^+ \approx 13.5$ at $y^+ = 30$.

As $u_b^+ = \sqrt{\frac{2}{f}}$, Eq. (15-8) give

$$\text{according to Einstein and Li } \overline{n}_w^+ = 0.785 \cdot \frac{f}{2} \quad (15-9)$$

$$\text{according to Hanratty } \overline{n}_w^+ = 4.3 \cdot 10^{-3} \quad (15-10)$$

Hanratty's result agrees with visual observations according to Table 2-2. Einstein and Li's result gives a lower frequency, because they assume that the whirls come from the bulk region whereby each whirl transfer more momentum. Besides, as the bulk region is involved, there will be a Reynold number dependence.

At $t = \theta$, Eqs. (15-3 and 6) give for the velocity profile

$$u^+(y^+, \theta) = u_o^+ \cdot \operatorname{erf} \frac{y^+}{4} \sqrt{\frac{\pi}{u_o^+{}^2}} \quad (15-11)$$

Hanratty assumed that the whirls came from a region about $y^+ = 30$ with the average dimensionless fluid velocity $u^+ \approx 13.5$. Equation (15-11) gives for this region

$$u^+ (30, \theta) = 13.5 \cdot 0.84$$

This means that the inner part of the fluid element is affected by the momentum transfer to the wall and thus one of the assumptions which lead to Eq. (15-3) is invalid. However, as the "a priori" predicted penetration frequency of Hanratty's model well agrees with experimental investigations, one must conclude that the model is reasonable and gives a fairly good description of the turbulence phenomena.

16. The Film-Penetration Model

Toor and Marchello (1958) considered that the whole fluid element was affected during the wall contact time. The fluid elements which reached the wall were assumed to come from a region of distance y_0 from the wall. The mass transfer between this region and the bulk region was assumed to be so rapid, that the concentration at $y = y_0$ was constant (c_0) all the time. Then the same differential equation as in chapter 14 is obtained (Eq. (14-2)), but one boundary condition is not the same

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2} \quad (16-1)$$

I.C.	$c = c_0$	at	$t = 0$
B.C. 1	$c = c_w$	at	$y = 0$
B.C. 2	$c = c_0$	at	$y = y_0$

The solution of Eq. (16-1) with initial and boundary conditions is found in Carslaw and Jaeger (1959) and will be

$$j_w = (c_0 - c_w) \sqrt{\frac{D}{\pi t}} \left[1 + 2 \sum_{i=1}^{\infty} \exp \left(-i^2 \pi^2 \frac{y_0^2}{Dt} \right) \right] \quad (16-2)$$

$$j_w = (c_0 - c_w) \frac{D}{y_0} \left[1 + 2 \sum_{i=1}^{\infty} \exp \left(-i^2 \pi^2 \frac{Dt}{y_0^2} \right) \right] \quad (16-3)$$

Equations (16-2 and 3) are identical. Equations (16-2) and (16-3) will

converge very rapidly at values of $\frac{Dt}{y_0^2}$ below and above one respectively.

If the contact time (θ) is assumed to be constant, integration of Eqs. (16-2 and 3) give approximatively

$$\bar{J}_W = 2 \cdot (c_0 - c_W) \sqrt{\frac{D}{\pi \theta}} \cdot (1 + 2\sqrt{\pi} \operatorname{ierfc} \frac{y_0}{\sqrt{D\theta}}) \quad \frac{D\theta}{y_0^2} < 1 \quad (16-4)$$

$$\bar{J}_W = (c_0 - c_W) \cdot \frac{D}{y_0} \cdot (1 + \frac{1}{3} \frac{y_0^2}{D\theta}) \quad \frac{D\theta}{y_0^2} > 1 \quad (16-5)$$

At low mass diffusivities Eq. (16-4) agrees with Higbie's penetration model. At high mass diffusivities Eq. (16-5) agrees with the film theory. This is because at high values of $\frac{D\theta}{y_0^2}$ the concentration profile is linear during most of the wall contact time.

If $\theta = \frac{1}{\bar{n}_w}$, one get

$$\frac{D\theta}{y_0^2} = \frac{1}{\bar{n}_w^+ \cdot y_0^{+2} \cdot Sc} \quad (16-6)$$

According to Table 2-2, $\bar{n}_w^+$ is about $5 \cdot 10^{-3}$. Then Eq. (16-6) gives that for $Sc = 1$ and $Sc = 10$, the value of $\frac{D\theta}{y_0^2}$ is below one if $y_0^+ > 15$ and

$y_0^+ > 5$ respectively. Thus the conclusion is that for turbulent duct flow at $Sc > 1$ the value of $\frac{D\theta}{y_0^2}$ is below one and the film penetration

model gives Eq. (16-4). At high Schmidt numbers the concentration boundary layer is so thin that $c_0 = c_b$, and then the film penetration model predicts at least as high mass transfer coefficients as Higbie's analysis, which predicts too high values. Thus for turbulent duct flow at $Sc > 1$ the film penetration model does not predict more reliable data than the simple penetration model.

17. Harriott's analysis

Experimental investigations have shown that all whirls do not reach the wall. Harriott (1962) considers this in his analysis, where both the whirl penetration depth and contact time are varying. At time $t = 0$ a whirl with bulk concentration penetrates to a distance δ from the wall.

It is assumed the whirl replaces all fluidum from $y = \delta$ to $y = \infty$. At $t = 0$ the concentration profile between the wall and $y = \delta$ is a function of earlier events.

The differential equation and initial and boundary conditions will be

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial y^2} \quad (17-1)$$

$$\text{I.C.} \quad c = c(y, 0) \quad \text{at} \quad t = 0$$

$$\text{B.C. 1} \quad c = c_w \quad \text{at} \quad y = 0$$

$$\text{B.C. 2} \quad c = c_b \quad \text{at} \quad y = \infty$$

The solution of this system is

$$c(y, t) = c_w \cdot \text{erfc} \sqrt{\frac{y^2}{4Dt}} + \frac{1}{\sqrt{4\pi Dt}} \int_0^\infty c(\psi, 0) \left(\exp\left(-\frac{(y-\psi)^2}{4Dt}\right) - \exp\left(-\frac{(y+\psi)^2}{4Dt}\right) \right) d\psi \quad (17-2)$$

$$c(y, 0) = c_b \quad \text{at} \quad y > \delta$$

At $y < \delta$, $c(y, 0)$ is obtained from Eq. (17-2) by iterative calculations.

The average mass transfer coefficient is obtained from

$$\bar{k} = \frac{1}{(c_b - c_w)} \cdot \frac{1}{t_\psi} \int_0^{t_\psi} D \left(\frac{\partial c}{\partial y} \right)_w dt_\psi \quad (17-3)$$

where the time t_ψ is long enough, so $\bar{k}$ does not vary with t_ψ . During the time t_ψ many whirls have succeeded each other.

Equations (17-2 and 3) are numerically solved by a computer.

Contact time and distance both constant

If the contact time (θ) and the penetration depth (δ) both are constant Eqs. (17-2 and 3) give

$$\bar{k} = 1.13 \sqrt{\frac{D}{\theta}} \quad \frac{D\theta}{\delta^2} > 10^2 \quad (17-4)$$

$$\bar{k} = \frac{D}{\delta} \quad \frac{D\theta}{\delta^2} < 10^{-2} \quad (17-5)$$

For small δ the whirls nearly reach the wall and the result (Eq. (17-4)) of course agrees with Higbie's analysis. For high δ most of the mass transfer resistance is in the film $0 < y < \delta$ and the result (Eq. (17-5)) agrees with the film theory. For intermediate values of $\frac{D\theta}{\delta^2}$ Eqs. (17-2 and 3) predicts a mass transfer coefficient which exceeds that predicted by Eq. (17-6) by only 0-5%

$$\frac{1}{k} = \frac{\delta}{D} + \frac{1}{1.13} \sqrt{\frac{\theta}{D}} \quad (17-6)$$

Variable contact time, constant distance

The penetration depth (δ) was constant while the contact time (θ) varied according to a certain distribution curve with average value $\bar{\theta}$. The resulting mass transfer coefficients agreed within 13% with the result obtained for constant contact time ($\theta = \bar{\theta}$). Thus the influence of the contact time distribution curve on the mass transfer coefficient is small.

Constant contact time, variable distance

The contact time (θ) was constant while the penetration depth (δ) varied according to two different distribution curves with the same average value $\bar{\delta}$. At low values of $\bar{\delta}$ ($\frac{D\theta}{\bar{\delta}^2} > 4$), i.e. when the whirls penetrate to a distance very near the wall, the resulting mass transfer coefficient was the same as with constant $\delta = \bar{\delta}$. At low values of $\frac{D\theta}{\bar{\delta}^2}$, however, the difference between constant and varying δ is remarkable. While constant δ predicts a result more like that of the film theory the lower the value of $\frac{D\theta}{\bar{\delta}^2}$, a varying δ predicts much higher values of the mass transfer coefficient. How high the mass transfer coefficient will be depends on the δ distribution curve chosen. The explanation for this big difference is that, for varying δ , even at high values of $\bar{\delta}$ a small fraction of the whirls will reach or nearly reach the wall. If then the mass molecular diffusivity is low, this small fraction of the whirls is of great importance for the mass transfer.

Harriott claims that his results agree well with experimental data for turbulent flow in circular ducts at $100 < Sc < 3000$. The assumptions are

$$\bar{\delta}^+ = 6$$

$$\bar{\theta}^+ / \nu = 16 \cdot 10^6 \text{ s/m}^2$$

and certain distribution curves for δ and θ . However, it is easy to get results which are consistent with experimental data, when there are several arbitrarily chosen parameters. Yet Harriott's analysis is most interesting, as it shows the great importance at high Schmidt numbers of the low frequency whirls which reach or nearly reach the wall.

18. The Penetration Boundary Layer Flow Model

Ruckenstein (1958) further developed the penetration theory by assuming that the whirls which reached the wall developed a boundary layer flow, which consists with the observations of Popovich and Hummel (1967). This generates a transverse velocity, so convective mass transfer to the wall exists even during the time between the whirl penetrations. Ruckenstein assumes that

- whirls leave a region with axial velocity u_0 and penetrate to the wall.
- the fluid element (the whirl) at the wall flows the distance x_0 and is then exchanged with a new whirl.
- the distance x_0 is so small that the inner parts of the fluid element are unaffected during the time at the wall.
- the Schmidt number is high enough, so in the region where the whirls come from $c = c_b$.

Properly treated this model results in very complicated mathematics, but Ruckenstein assumed that the result will be about the same as for stationary flow along a flat plate with length x_0 . The two-dimensional boundary layer equations for this system become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (\text{continuity}) \quad (18-1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (\text{motion}) \quad (18-2)$$

$$u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \quad (\text{continuity of the solute}) \quad (18-3)$$

The boundary conditions are

$$\begin{array}{llllll} \text{B.C. 1} & u = u_0 & \text{and} & c = c_b & \text{at} & x = 0 \\ \text{B.C. 2} & u = v_y = 0 & \text{and} & c = c_w & \text{at} & y = 0 \\ \text{B.C. 3} & u = u_0 & \text{and} & c = c_b & \text{at} & y = \infty \end{array}$$

The boundary layer theory (e.g. Bird et al. (1960, Chapter 11.4)) is used to solve these equations. The same result will be obtained if the exact solution according to Bird et al. (1960, Chapter 19.3) is used. The following velocity and concentration profiles are assumed

$$\frac{u}{u_0} = 2 \frac{y}{\delta_m} - 2 \left(\frac{y}{\delta_m} \right)^3 + \left(\frac{y}{\delta_m} \right)^4 \quad (18-4)$$

$$\frac{c - c_w}{c_b - c_w} = 2 \frac{y}{\delta_q} - 2 \left(\frac{y}{\delta_q} \right)^3 + \left(\frac{y}{\delta_q} \right)^4 \quad (18-5)$$

δ_m and δ_q are functions of x and denote the boundary layer thickness of momentum and mass respectively.

Equations (18-1) to (18-5) give

$$\delta_m = \sqrt{\frac{1260}{37} \frac{\nu x}{u_0}} \approx 5.83 \sqrt{\frac{\nu x}{u_0}} \quad (18-6)$$

$$\delta_q = \delta_m \cdot Sc^{-\frac{1}{3}} \quad (18-7)$$

$$k = \frac{D \frac{\partial c}{\partial y} \big|_w}{c_b - c_w} = 0.34 \cdot D \left(\frac{u_0}{\nu x} \right)^{\frac{1}{2}} \cdot Sc^{\frac{1}{3}} \quad (18-8)$$

$$\bar{k} = \frac{1}{x_0} \int_0^{x_0} k dx = 0.68 \cdot D \left(\frac{u_0}{\nu x_0} \right)^{\frac{1}{2}} \cdot Sc^{\frac{1}{3}} \quad (18-9)$$

Ruckenstein (1967) uses the same method as Hanratty (1956) and Einstein and Li (1956) used to calculate x_0 .

$$\bar{\tau}_w = \mu \frac{1}{x_0} \cdot \int_0^{x_0} \frac{\partial u}{\partial y} \big|_w dx \quad (18-10)$$

Equations (18-4, 6 and 10) give

$$\bar{\tau}_w = 0.68 \cdot u_0 \cdot \mu \left(\frac{u_0}{\nu x_0} \right)^{\frac{1}{2}} \quad (18-11)$$

Equations (18-9 and 11) and $u^* = \sqrt{\frac{\tau_w}{\rho}}$ give

$$\bar{k} = \frac{u^*}{u_0^+} \cdot Sc^{-\frac{2}{3}} \quad (18-12)$$

Ruckenstein assumes that u_0^+ is in the order of ten. This agrees with previously mentioned experimental investigations in chapter 2, and if it is assumed that $u_0^+ = 13.5$ according to Hanratty (1956), Eq. (18-12) becomes

$$\bar{k} = 0.074 \cdot u^* \cdot Sc^{-\frac{2}{3}} \quad (18-13)$$

In dimensionless form Eq. (18-13) becomes

$$Sh = 0.074 \cdot \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{\frac{1}{3}} \quad (18-14)$$

Equation (18-14) is shown in Diagram 1c, and it is evident that the agreement with experimental data is fairly good. At Schmidt numbers in the order of one the concentration in the region where the whirls come from is not the bulk value. This is one reason why Eq. (18-14) overpredicts the mass transfer coefficient at low Schmidt numbers.

The whirl frequency is calculated from Eq. (18-11) and $\bar{n} = \frac{u_0}{x_0}$.

$$\bar{n}^+ = \frac{2.16}{u_0^{+2}} \quad (18-15)$$

For $u_0^+ = 13.5$ Eq. (18-15) gives $\bar{n}^+ = 11.9 \cdot 10^{-3}$ which is higher than experimental investigations according to Table 2-2.

Ruckenstein's model assumes that all whirls reach the wall and gives a too high whirl frequency. Yet the resulting mass transfer coefficient agrees well with experimental data. This is because Ruckenstein assumes stationary conditions. Practically, when the whirls reach the wall unsteady conditions must prevail at least in the beginning of the contact

time. During the early unsteady conditions the mass and momentum transfer rates are much higher than during steady conditions, so for each wall penetrating whirl Ruckenstein's calculations give too low mass and momentum transfer. This is balanced with the too high whirl frequency.

19. Pinczewski and Sideman's Analysis

Pinczewski and Sideman (1974) used Ruckenstein's physical model, but they considered unsteady boundary layer flow. Then the two-dimensional boundary layer equations become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (19-1)$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \quad (19-2)$$

$$\frac{\partial c}{\partial t} + u \frac{\partial c}{\partial x} + v \frac{\partial c}{\partial y} = D \frac{\partial^2 c}{\partial y^2} \quad (19-3)$$

The initial and boundary conditions are

I.C.	$u = u_0$	and	$c = c_b$	at	$t = 0$
B.C. 1	$u = u_0$	and	$c = c_b$	at	$x = 0$
B.C. 2	$u = v_y = 0$	and	$c = c_w$	at	$y = 0$
B.C. 3	$u = u_0$	and	$c = c_b$	at	$y = \infty$

The difference between the Eqs. (19-1, 2 and 3) and (18-1, 2 and 3) are the time dependent terms in Eqs. (19-2 and 3).

Rozenshtok (1965) solved the boundary layer equations (19-1, 2 and 3) with their initial and boundary conditions. He assumed the same velocity and concentration profiles as in Eqs. (18-4 and 5) and at $Sc > 1$ he obtained

$$\delta_q = 5.72 \sqrt{\frac{\nu x}{u_0}} \cdot Sc^{-\frac{1}{3}} \quad x < x_{is} \quad (19-4)$$

$$\delta_q = 3.65 \sqrt{\nu t} \cdot Sc^{-\frac{1}{2}} \quad x > x_{is} \quad (19-5)$$

$$x_{is} = 0.4 \cdot u_0 \cdot t \cdot Sc^{-\frac{1}{3}} \quad (19-6)$$

x_{is} is the distance where the concentration boundary layer changes from a steady to an unsteady one. At $t = 0$ unsteady conditions prevail in the whole boundary layer. Then steady conditions prevail for a distance increasing with time (Eq. (19-6)). Of course the boundary layer thickness during steady conditions (Eq. (19-4)) agrees with Ruckenstein's result (Eq. (18-7)). As $k = \frac{D}{c_b - c_w} \left. \frac{\partial c}{\partial y} \right|_w$, Eqs. (18-5) and (19-4 and 5) give

$$k_s = 0.35 D \cdot \left(\frac{u_0}{xv} \right)^{\frac{1}{2}} \cdot Sc^{\frac{1}{3}} \quad x < x_{is} \quad (19-7)$$

$$k_{is} = 0.55 \sqrt{\frac{D}{t}} \quad x > x_{is} \quad (19-8)$$

Equation (19-8) is not quite correct. Between x_{is} , which is the length of the steady concentration boundary layer, and $x = 0.4 \cdot u_0 t$, which is the length of the steady momentum boundary layer, the mass transfer coefficient is somewhat lower than that predicted by Eq. (19-8). However, this difference is negligible. Not surprisingly the steady result (Eq. (19-7)) agrees with Ruckenstein's result (Eq. (18-8)). Perhaps surprisingly, the unsteady mass transfer coefficient (Eq. (19-8)) will be the same as that obtained from Higbie's analysis (Eq. (14-5)).

Then Pinczewski and Sideman calculate the average mass transfer coefficient $\bar{k}$ by

$$\bar{k} = \frac{1}{x_\theta} \left(\int_0^{0.4 \cdot x_\theta \cdot Sc^{-\frac{1}{3}}} k_s dx + \int_{0.4 \cdot x_\theta \cdot Sc^{-\frac{1}{3}}}^{x_\theta} k_{is} dx \right) \quad (19-9)$$

$$x_\theta = u_0 \cdot \theta$$

Thus they calculate the average k over the distance x_θ at the moment when the fluid element at the wall is replaced with a fresh fluid element (whirl). This average value is illogically assumed to be the total average value. Either the authors have made a logical mistake, or a printing mistake. In any event their result became

$$\bar{k} = \sqrt{\frac{v u_0}{x_0}} \cdot Sc^{-\frac{1}{2}} \cdot (1.1 + 0.44 Sc^{-\frac{1}{3}} - 0.7 Sc^{-\frac{1}{6}}) \quad (19-10)$$

The quotient $\frac{u_0}{x_0}$ is equal to the penetration frequency n . Pinczewski and Sideman use $n^+ = 4.1 \cdot 10^{-3}$ from experimental investigations. However they assume that only 7% of the whirls penetrate to the wall. The other 93% penetrate to a distance $y^+ = 1$ from the wall.

$$\underline{1 < Sc < 10}$$

At low Schmidt numbers the mass molecular diffusivity is high enough, so the mass transfer resistance between the wall and $y^+ = 1$ can be neglected. Then Eq. (19-20) and $n^+ = 4.1 \cdot 10^{-3}$ give

$$\bar{k} = 0.064 \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{-\frac{1}{2}} \cdot (1.1 + 0.44 Sc^{-\frac{1}{3}} - 0.70 Sc^{-\frac{1}{6}}) \quad (19-11)$$

$$\underline{10 < Sc < 10^3}$$

At higher Schmidt number the mass transfer resistance between the wall and $y^+ = 1$ can not be neglected. It is assumed that the film theory is valid here. The additional mass transfer rate of those whirls which penetrate to the wall is neglected. Then the result is

$$\bar{k} = \frac{0.064 \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{-\frac{1}{2}} \cdot (1.1 + 0.44 Sc^{-\frac{1}{3}} - 0.70 Sc^{-\frac{1}{6}})}{1 + 0.064 Sc^{\frac{1}{2}} \cdot (1.1 + 0.44 Sc^{-\frac{1}{3}} - 0.70 Sc^{-\frac{1}{6}})} \quad (19-12)$$

$$\underline{Sc > 10^3}$$

At high Schmidt numbers Pinczewski and Sideman do not use their own model, probably because its result does not agree with experimental data. Instead a simplified version of Ruckenstein's analysis is used and after doubtful assumptions the result is

$$k = 0.067 \sqrt{\frac{f}{2}} \cdot u_b \cdot Sc^{-\frac{2}{3}} \quad (19-13)$$

Pinczewski and Sideman's result, Eqs. (19-11, 12 and 13) is shown in Diagram 1c and it is evident that the agreement with experimental data

is good. However, to obtain this good agreement they have mistreated their own model. The right way to calculate the average mass transfer coefficient from Eqs. (19-7 and 8) is to integrate over both the time and the distance

$$\bar{k} = \frac{1}{\theta \cdot x_{\theta}} \cdot \int_0^{\theta} \int_0^{x_{\theta}} k \, dx \, dt \quad (19-14)$$

$$\bar{k} = \frac{1}{\theta \cdot x_{\theta}} \cdot \int_0^{\theta} \left[\int_0^{x_{\theta}} \left(0.4 \cdot u_0 \cdot t \cdot Sc^{-\frac{1}{3}} + 0.35 \cdot D \cdot \left(\frac{u_0}{v_x} \right)^{\frac{1}{2}} \cdot Sc^{\frac{1}{3}} \right) dx + \int_0^{x_{\theta}} \left(0.55 \cdot \left(\frac{D}{t} \right)^{\frac{1}{2}} \right) dx \right] dt - \frac{0.04 \cdot u_0 \cdot t \cdot Sc^{-\frac{1}{3}}}{\theta \cdot x_{\theta}} \quad (19-15)$$

Equation (19-15) and $x_{\theta} = u_0 \cdot \theta$ give

$$\bar{k} = \sqrt{\frac{D}{\theta}} \cdot (1.10 + 0.15 \, Sc^{-\frac{1}{3}}) \quad (19-16)$$

The difference between Eq. (19-16) and the result of Pincewski and Sideman (Eq. (19-10)) is inside the parentheses and is small at high Schmidt numbers. The difference between Eq. (19-16) and the result of Higbie's analysis (Eq. (14-7)) is small, 10% at $Sc = 1$ and below 3% at $Sc > 15$. Thus if the whirls are assumed to reach the wall the unsteady penetration boundary layer flow model gives about the same result as Higbie's simple penetration model.

20. HUGMARK'S ANALYSIS

Hughmark deals with turbulent mass, heat and momentum transfer in a series of papers (1969, 1971a, 1971b, 1972, 1973, 1975 and 1977). Finally he, like von Kármán, divides the cross sectional area into three regions

wall region	(wr)	$0 < y^+ < 1.6$
intermediate region	(ir)	$1.6 < y^+ < 34.6$
core region	(cr)	$y^+ > 34.6$

The boundaries $y^+ = 1.6$ and $y^+ = 34.6$ are chosen according to the result of Popovich and Hummel (1967). In contrast to von Kármán, Hughmark considers both turbulent and molecular mass transfer in all regions. As the peripheral areas at $y^+ = 1.6$ and $y^+ = 34.6$ are about the same as that at the wall, Eq. (20-1) is independent of the wall curvature.

$$\frac{1}{k} = \frac{1}{k_{wr}^{(m)} + k_{wr}^{(t)}} + \frac{1}{k_{ir}^{(m)} + k_{ir}^{(t)}} + \frac{1}{k_{cr}^{(m)} + k_{cr}^{(t)}} \quad (20-1)$$

The superscripts (m) and (t) denote the molecular and the turbulent part of the quantity respectively. k_{wr} , k_{ir} and k_{cr} are the mass transfer coefficients between the inner and outer boundary of respective regions and are based on the outer peripheral area of the region.

Molecular transfer

For the core region Hughmark uses the velocity profile for turbulent flow in circular ducts according to Bogue and Metzner (1963). With the assumption $\epsilon_q = \epsilon_m$ he obtained by numerical integration at different mass diffusivities.

$$k_{cr}^{(m)} = 7.16 \frac{D}{d_h} \quad (20-2)$$

This equation is only valid in cylindrical geometry. It is interesting to note that Eq. (20-2) is very similar to the local mass transfer coefficient for laminar flow with developed concentration profile in circular ducts at constant wall mass flux. In this case the concentration profile will be (Bird et al. (1960, Chapter 9.8)).

$$c - c_w = \left[\frac{3}{4} - \left(\frac{2r}{d_h} \right)^2 + \frac{1}{4} \left(\frac{2r}{d_h} \right)^4 \right] \cdot \frac{j_w \cdot d_h}{2 \cdot D} \quad (20-3)$$

By integration over the cross sectional area, Eq. (20-3) and the laminar velocity profile give

$$c_b - c_w = \frac{11}{48} \frac{j_w \cdot d_h}{D} \quad (20-4)$$

$$k = \frac{j_w}{c_b - c_w} = 4.36 \frac{D}{d_h} \quad (20-5)$$

In the intermediate and wall regions Hughmark uses the film theory

$$j^{(m)} = D \frac{\Delta c}{\Delta y} \quad (20-6)$$

$$k^{(m)} = \frac{j^{(m)}}{\Delta c} = \frac{D}{\Delta y} \quad (20-7)$$

By definition $\Delta y = \Delta y^+ \cdot \frac{\nu}{u^*}$ which gives

$$k_{ir}^{(m)} = \frac{u^*}{33 \cdot Sc} \quad (20-8)$$

$$k_{wr}^{(m)} = \frac{u^*}{1.6 \cdot Sc} \quad (20-9)$$

Equations (20-6) to (20-9) are valid in laminar flow near the wall and presumably also in turbulent flow if the concentration gradient is constant within the region. However this is not always the case, as the concentration gradient varies considerably within a region. The position of this region is dependent of the Schmidt number. However, if the turbulent mass transfer dominates over the molecular mass transfer in this region, it does not matter if the molecular mass transfer equations are erroneous.

Turbulent transfer

For the core reigon Hughmark, from empricial data, evaluates the sum of the turbulent and molecular mass transfer coefficient. Then he substracts the molecular mass transfer coefficient (Eq. 20-2)) and obtains

$$k_{cr}^{(t)} = f \cdot u_b \quad (20-10)$$

Equation (20-10) can also be obtained from Reynolds analogy $\Delta c^+ = \Delta u^+$.

According to definition and the fact that $j \approx j_w$ at the outer part of the core region

$$k_{cr} = \frac{u^*}{\Delta c^+}, \text{ which give}$$

$$k = \frac{f \cdot u_b^2}{2 \cdot \Delta u} \quad (20-11)$$

If the fluid velocity at $y^+ = 34.6$ is half the bulk velocity, which it very roughly is, Eqs. (20-10) and (20-11) are identical.

For the intermediate region, Hughmark assumes that Higbie's penetration model is valid. The dimensionless penetration frequency n^+ is assumed to be $3 \cdot 10^{-3}$ and then Higbie's Eq. (14-7) gives

$$k_{ir}^{(t)} = 0.062 \cdot u^* \cdot Sc^{-\frac{1}{2}} \quad (20-12)$$

For the wall region, Hughmark adapts the correlation for the turbulent mass transfer coefficient so Eq. (20-1) will be consistent with experimental data. The resulting correlation is

$$k_{wr}^{(t)} = 0.047 \cdot u^* \cdot Sc^{-\frac{2}{3}} \quad (20-13)$$

Equation (20-13) can also be obtained from Ruckenstein's analysis by setting $u_0^+ = 21$ in Eq. (18-12).

Equation (20-1) can now be written

$$\frac{1}{k} = \frac{1}{\frac{u^*}{1.6 Sc} + 0.047 \cdot u^* \cdot Sc^{-\frac{2}{3}}} + \frac{1}{\frac{u^*}{33 Sc} + 0.062 \cdot u^* \cdot Sc^{-\frac{1}{2}}} + \frac{1}{7.16 \frac{D}{d_h} + f \cdot u_b} \quad (20-14)$$

Equation (20-14) is valid for turbulent flow in circular ducts, but also for other geometries if the core region molecular mass transfer is negligible. Equation (20-14) in dimensionless form is shown in Diagram 1d, and it is evident that the agreement with experimental data is very good. Of course one reason for this is that several constants are empirically determined.

To conclude Hughmark has derived a useful correlation for the mass transfer coefficient, which is based on

- laminar flow theory for prediction of the molecular mass transfer rate
- Reynolds analogy for prediction of the turbulent mass transfer rate in the core region
- Higbie's penetration theory for prediction of the turbulent transfer rate in the intermediate region
- Ruckenstein's penetration boundary layer flow model for prediction of the turbulent transfer rate in the wall region.

Hughmark (1977) uses Eq. (20-14) to determine the molecular part of the total mass transfer rate. However, these results are not reliable as the correlations for the molecular mass transfer coefficients of the wall and intermediate regions are sometimes erroneous, as mentioned above.

21 CONCLUSIONS, EMPIRICAL CORRELATIONS AND RECOMMENDATIONS

In turbulent flow mass is transferred both by molecular diffusion and by whirls in three dimensions. The whirls are irregular in time and space so the local concentration and mass transfer rate vary in time. It is impossible to analytically predict these time fluctuations. The above described models try to determine the time average values, but as the models are not quite truthful, their results are just approximate. By comparing the results of the models with experimental data, the following conclusions are made.

- In the core region at $Sc > 1$ Reynolds analogy is valid, which among others means that the molecular mass transfer rate is negligible.
- In a transition region there is rapid whirl exchange, so there the unsteady penetration model is applicable.
- In the wall region the whirl penetration frequency is very low, so there mainly intermittent stationary boundary layer flow prevails.

The most realistic of the models described in this work is probably that of Harriott (1962), where both the contact time and the penetration depth vary. However this model is not practically useful. Certainly it is possible to further improve Harriott's model by assuming steady boundary layer flow in the layer between the wall and the penetrating whirl. However in addition to the other models this eventual improvement brings nothing new to the understanding of the turbulence phenomena near a solid wall, and still several parameters have to be empirically estimated.

As there is no exact mathematical model for the turbulent mass transfer, many purely empirical correlations have been presented. All models described in this work have resulted in

$$Sh = Sh(Re, Sc)$$

Then several empirical correlations have been in the form

$$Sh = a_1 \cdot Re^{a_2} \cdot Sc^{a_3} \quad (21-1)$$

where a_i are constants. Examples are

$$Sh = 0.0243 \cdot Re^{0.8} \cdot Sc^{0.4} \quad (\text{Dittus-Boelter})$$

$$Sh = 0.023 \cdot Re^{0.8} \cdot Sc^{0.4} \quad (\text{McAdams})$$

$$Sh = 0.023 \cdot Re^{0.8} \cdot Sc^{\frac{1}{3}} \quad (\text{Colburn})$$

These are shown in Diagram 1e, and it is evident that it is not possible to correlate the Sherwood number over the whole Schmidt number range with constant Schmidt number exponent. The correlations of Dittus-Boelter and McAdam are fairly consistent with experimental data at $0.5 < Sc < 10^3$ while Colburn's correlation is fairly consistent with experimental data at $Sc > 10^3$. It should be observed that the often used correlation of Deissler is not good either (see diagram 1b).

$$Sh = 0.112 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{\frac{1}{4}} \quad Sc > 100 \quad (\text{Deissler}) \quad (21-2)$$

The general analysis of Reichardt resulted in

$$Sh = \frac{\frac{f}{2} \cdot Re \cdot Sc \cdot \frac{u_{\text{quot}}}{c_{\text{quot}}} \left\langle \frac{\varepsilon_q}{\varepsilon_m} \right\rangle}{1 + \xi + (Sc-1) \cdot b(Re, Sc) \cdot u_{\text{quot}} \cdot \sqrt{\frac{f}{2}}} \quad (\text{Reichardt}) \quad (21-3)$$

This is a more complicated form of correlation for the Sherwood number, which has good possibilities in correlating experimental data over a broad range of Schmidt and Reynolds numbers. Petukhov (1970) did a similar analysis and numerically calculated $Sh = Sh(Re, Sc)$. He determined some parameters from experimental data. The correlation below correlated calculated data within 10% at $0.5 < Sc < 10^3$

$$Sh = \frac{\frac{f}{2} \cdot Re \cdot Sc}{1.07 + 12.7 \sqrt{\frac{f}{2}} \cdot (Sc^{\frac{2}{3}} - 1)} \quad \left\{ \begin{array}{l} 10^4 < Re < 5 \cdot 10^6 \\ Sc > 0.5 \end{array} \right. \quad (\text{Petukhov}) \quad (21-4)$$

At $Sc > 10^3$ the equation can be written as

$$Sh = 0.079 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{\frac{1}{3}} \quad (21-5)$$

Petukhov's correlation is shown in Diagram 1e, and it is evident that the agreement with experimental data is good at $0.5 < Sc < 10^3$. At higher Schmidt numbers Petukhov's correlation predicts too high mass transfer coefficients, because of the experimental data he used to determine some parameters. In fact Petukhov himself proposes a still higher constant than 0.079 at $Sc > 2 \cdot 10^3$.

As earlier presented, Notter and Sleicher (1972) obtained

$$Sh = 5 + 0.015 Re^a Sc^b \quad \begin{cases} 10^4 < Re < 10^6 \\ 0.1 < Sc < 10^4 \end{cases} \quad (\text{Notter-Sleicher}) \quad (21-6)$$

$$a = 0.88 - \frac{0.24}{4+Sc}$$

$$b = \frac{1}{3} + 0.5 \exp(-0.6 \cdot Sc)$$

Thus the Reynolds and Schmidt numbers exponents varies with the Schmidt number. At $Sc > 0.5$ at $Re = 10^4$ the correlations of Petukhov and Notter-Sleicher are consistent within 3%. The maximum difference between these two correlations is at low Schmidt numbers and high Reynolds numbers. At $Sc=1$ and $Re=10^6$ they differ about 10% from each other.

Churchill (1977) proposes the following correlation at all Schmidt numbers and $Re > 10^4$

$$Sh = Sh_0 + \frac{0.079 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc}{(1 + Sc^{\frac{4}{5}})^{\frac{5}{6}}} \quad \left\{ Re > 10^4 \quad (\text{Churchill}) \right. \quad (21-7)$$

$$Sh_0 \simeq 4.8 \text{ to } 6.3$$

At $Sc > 3$ and $Re = 10^4$ the correlations of Petukhov and Churchill are consistent within 3%. However at $Sc < 10$ and $Re > 10^5$ the correlations of Notter and Sleicher, Churchill and Petukhov give different results. At $Re=10^6$ the maximum difference is about 20%, and constantly Churchill's correlation gives the highest Sherwood number. At high Reynolds number Churchill's correlation predicts $k = \sqrt{\frac{f}{2}} \cdot u_b \cdot k(Sc)$, which is the result if the shear stress is practically constant in the concentration boundary layer. This will be the case for the lower Schmidt number the higher the Reynolds number, but will probably never be the case for as low Schmidt number as one. Therefore the correlation of Churchill probably is not as reliable as those of Petukhov and Notter and Sleicher.

At high Schmidt numbers the whole mass transfer resistance is near the wall. Then even very small wall roughness will strongly influence the mass transfer. In practice the wall is never completely smooth, which is probably the reason for the scattering of experimental mass transfer

data at high Schmidt numbers. This is not seen in Diagram 1, as at high Schmidt number only those experimental data are presented which are obtained from experiments with comparatively smooth walls. The most reliable smooth wall data are probably those of Shaw and Hanratty (1977a), who experimentally obtained the following correlation

$$Sh = 0.0889 \cdot \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{0.296} \quad 700 < Sc < 37000 \quad (21-8)$$

Equation (21-8) agrees with the result from the dimensional analysis, Eq. (7-13), and is shown in Diagram 1d.

The conclusion is that at $Sc > 1$ and the following conditions

- smooth walls
- fully developed flow in circular ducts
- $Re > 10^4$
- constant physical properties
- nonpermeable walls

the correlation of Hughmark (Eq. (20-14)) is valid over the whole Schmidt number range. However this equation is rather inconvenient to use, so at $Sc < 300$ the correlations of Petukhov (Eq. (21-4)) and Notter-Sleicher (Eq. (21-6)) and at $Sc > 700$ the correlation of Shaw-Hanratty (Eq. (21-8)) are recommended. At $300 < Sc < 700$ the above correlations overpredict and underpredict respectively the Sherwood number, so there the correlations of Deissler (Eq. (10-9)) and Ruckenstein (Eq. (18-14)) are recommended.

22. ROUGH WALLS

Wall roughness increases the momentum transfer for three reasons

- the wall surface area is increased
- the turbulence near the wall is increased because of increased disturbance of the velocity field near the wall
- the pressure is unsymmetrical around the roughness elements, which produces the "form drag"

Which way the mass transfer is influenced of the wall roughness depends on the nature of the roughness elements. If they have the same intrinsic mass transfer resistance as the primary wall, the increased wall surface area and turbulence near the wall will also increase the mass transfer rate. However the mass transfer is not influenced by the "form drag". If the roughness elements themselves are not involved in the mass transfer probably quite different conditions will prevail. First the wall surface area active for mass transfer is decreased. Second the increase in the turbulence mainly influences the upper parts of the roughness elements, and perhaps the turbulence at the lower parts of the roughness elements and the primary wall is decreased by the roughness. Thus this kind of roughness may decrease the mass transfer rate. However, in this work only that kind of roughness is considered, which gives the same intrinsic mass transfer resistance as the wall.

Momentum Transfer

According to Dalle Donne and Meyer (1977), Nikuradse was the first to carefully examine the momentum transfer dependence on the wall roughness (published 1933). He glued rather homogenous size fractions of sand particles as closely to each other as possible to the walls of circular ducts and measured the resulting fluid velocity profile. For smooth walls the fluid velocity profile for $y^+ > 70$ will be approximately

$$u^+ = 2.5 \ln y^+ + 5.5 \quad \begin{cases} \text{smooth wall} \\ y^+ > 70 \end{cases} \quad (22-1)$$

Nikuradse got for his rough walls

$$u^+ = 2.5 \ln y^+ - 2.5 \ln e^+ + u_e^+(e^+) \quad y^+ > 70 \quad (22-2)$$

e is the height of the sand particles and e^+ is the dimensionless height $e^+ = e \cdot u^*/\nu$. The dimensionless velocity profile for a rough wall with a given surface roughness differs only by an additive factor from that of a smooth wall. Physically $u_e^+(e^+)$ can be regarded as the dimensionless fluid velocity at the top of the sand particles (for $y^+ = e^+$ Eq. (22-2) give $u^+ = u_e^+(e^+)$).

The fanning friction factor (f) can be obtained from Eq. (22-2) by integrating both sides over the cross sectional area. Of course Eq. (22-2) is not valid near the wall ($y^+ < 70$), but as the fluid velocity there is so low, the error introduced is negligible

$$\frac{1}{y_c^{+2}} \int_0^{y_c^+} u^+ \cdot 2 \cdot r^+ dr^+ = u_b^+ = 2.5 \ln \frac{y_c}{e} - 3.7 + u_e^+(e^+) \quad (22-3)$$

$$y^+ = y_c^+ - r^+$$

According to definition $u_b^+ = \frac{1}{\sqrt{f/2}}$, which together with Eq. (22-3) give

$$\sqrt{\frac{f}{2}} = \frac{1}{2.5 \ln \frac{y_c}{e} - 3.75 + u_e^+(e^+)} \quad (22-4)$$

Nikuradse obtained the following correlation for the velocity term $u_e^+(e^+)$

$$e^+ < 5 \quad u_e^+(e^+) = 5.5 + 2.5 \ln e^+ \quad (22-5)$$

Thus the velocity profile will be the same as for the smooth wall. This means that the momentum transfer is not influenced by sand particles on the wall with dimensionless height less than 5, i.e. the friction factor is the same as for smooth walls.

$5 < e^+ < 55-70$. In an intermediate region of e^+ , by increasing e^+ , $u_e^+(e^+)$ first begins to increase more slowly than predicted by Eq. (22-5) and then at e^+ between 10 and 20, $u_e^+(e^+)$ reaches its maximum value and then begins to decrease with increasing e^+ until a constant value is reached at e^+ about 70. At all values of e^+ , $8.5 < u_e^+(e^+) < 10$. According to Eq. (22-4), by increasing e^+ (the same as increasing Reynolds number at constant $\frac{e}{d_h}$) the friction factor will first decrease and then

increase until it reaches a constant value.

$$f = f \left(\frac{e}{d_h}, e^+ \right)$$

$$e^+ > 55-70 \quad u_e^+ (e^+) = 8.5$$

According to Eq. (22-4) the friction factor is only dependent on $\frac{e}{d_h}$, $f = f \left(\frac{e}{d_h} \right)$. Thus the friction factor is independent of the Reynolds number and thus also of the kinematic viscosity. This means that the molecular transfer is negligible at high values of e^+ .

Nikuradse's result is valid for closely packed particles, such as sand particles, on solid walls. For other roughness geometries the result will be similar but not necessarily the same. At small e^+ the friction factor is unaffected by the roughness. The limit for this is probably about $e^+=5$ for all geometries of the roughness. At high e^+ the friction factor only depends on $\frac{e}{d_h}$, but the value of the term $u_e^+ (e^+)$ varies with different geometries of the roughness. Often Eq. (22-4) together with $u_e^+ (e^+) = 8.5$ at high e^+ , are used to define an equivalent sand roughness diameter for all kinds of roughness. However this equivalent sand roughness diameter is only applicable to momentum transfer for high e^+ where the friction factor is constant.

To conclude, the correlation for the friction factor at rough walls will be

$$f = f \left(e^+, \frac{e}{d_h}, \text{roughness geometry} \right)$$

Mass Transfer

Theoretical analysis

For mass transfer at rough walls and Schmidt number about 1-15 the reader is referred to Dipprey and Sabersky (1963), Kolar (1965), Gowen and Smith (1968), Dalle Donne and Meyer (1977) and Han et al. (1978).

When dealing with mass transfer at high Schmidt numbers and rough walls there are some problems in defining the surface $y=0$ and the wall shear stress. Usually $y=0$ is defined as that surface which gives the same volume per length unit as for the actual rough wall. This definition is not always practical at high Schmidt numbers, because then the whole concentration boundary layer might be below $y=0$ for most of the wall. In fact, in this work it is not important how $y=0$ is defined so for simplicity $y=0$ is here defined as above. The wall shear stress, as at smooth walls, is defined according to

$$\tau_w = \frac{\partial P}{\partial x} \cdot \frac{A}{S}$$

where S is the peripheral length at $y=0$.

It is impossible to analytically treat the turbulent mass and momentum transfer phenomena very near a rough wall. Instead dimensional analysis will be used here. At high Schmidt numbers the whole concentration boundary layer is so thin, that within it the shear stress is only dependent on the wall shear stress (τ_w) and the height and geometry of the roughness elements. Further the eddy diffusivity and the mass transfer coefficient is dependent on the physical properties of the fluid (ρ, ν, D). Hence the result is

$$k = k(\tau_w, \rho, \nu, D, e, \text{roughness geometry})$$

According to Buckingham's second π -theorem a correlation for the mass transfer coefficient then needs 4 dimensionless groups. The following four are chosen

$$\frac{k}{\sqrt{\frac{\tau_w}{\rho}}} \equiv \frac{k}{u^*} \equiv k^+$$

$$\frac{e \cdot u^*}{\nu} \equiv e^+$$

Sc

roughness geometry

Thus

$$k^+ = k(Sc, e^+, \text{roughness geometry})$$

(22-6)

For smooth walls correlation (22-6) gives $k^+ = k(Sc)$, which is consistent with the results at high Schmidt numbers ($Sc > 500$) of, among others, Shaw and Hanratty (1977a), Pethukov (1970) and Notter and Sleicher (1972).

Experimental investigations

Dawson and Trass (1972) experimentally examined the influence of rough walls on the mass transfer coefficient at high Schmidt numbers. They used an electrochemical technique in a square duct and the roughness elements were -shaped grooves running across the flow direction. The ratio of groove pitch to height was held as constant as possible. Their theoretical treatment of the problem was not satisfactory. They thought that the correlation for the mass transfer coefficient should be

$$\frac{k}{k_{e=0}} = k(Sc, e^+, \text{roughness geometry}) \quad (22-7)$$

However their experimental results did not agree with correlation (22-7), but $\frac{k}{k_{e=0}}$ was also dependent on $\frac{e}{d_h}$.

From Fig. 8 and Figs. 10-12 in the paper of Dawson and Trass it is possible to calculate k^+ as a function of Sc and e^+ .

$$k^+ = \frac{Sh}{Re \cdot Sc \cdot \sqrt{\frac{f}{2}}} \quad (22-8)$$

$$e^+ = \frac{e}{d_h} \cdot Re \cdot \sqrt{\frac{f}{2}} \quad (22-9)$$

This is done here and the result is shown in Fig. 22-1 together with some other results from Dawson and Trass (1972) in Fig. 22-1b. All points other than those represented by x and $+$ are from roughness with scale consistent geometry and six different heights ($2.2 < \frac{e}{d_h} \cdot 10^3 < 13.6$). As seen in Fig. 22-1, for each Schmidt number all these points are well represented by a single curve, apart from those for the lowest Reynolds number ($Re = 15000$). The reason for this deviation is either that the correlation (22-6) is not valid for $Re < 2 \cdot 10^4$ at $Sc < 10^3$, or is due to an experimental error. Dawson and Trass used a very rough method to determine the friction factor and did not consider that the wall shear stress varies over the wall (see Chapter 25). Further they did not use any entrance region for development of the concentration profile. These conditions will

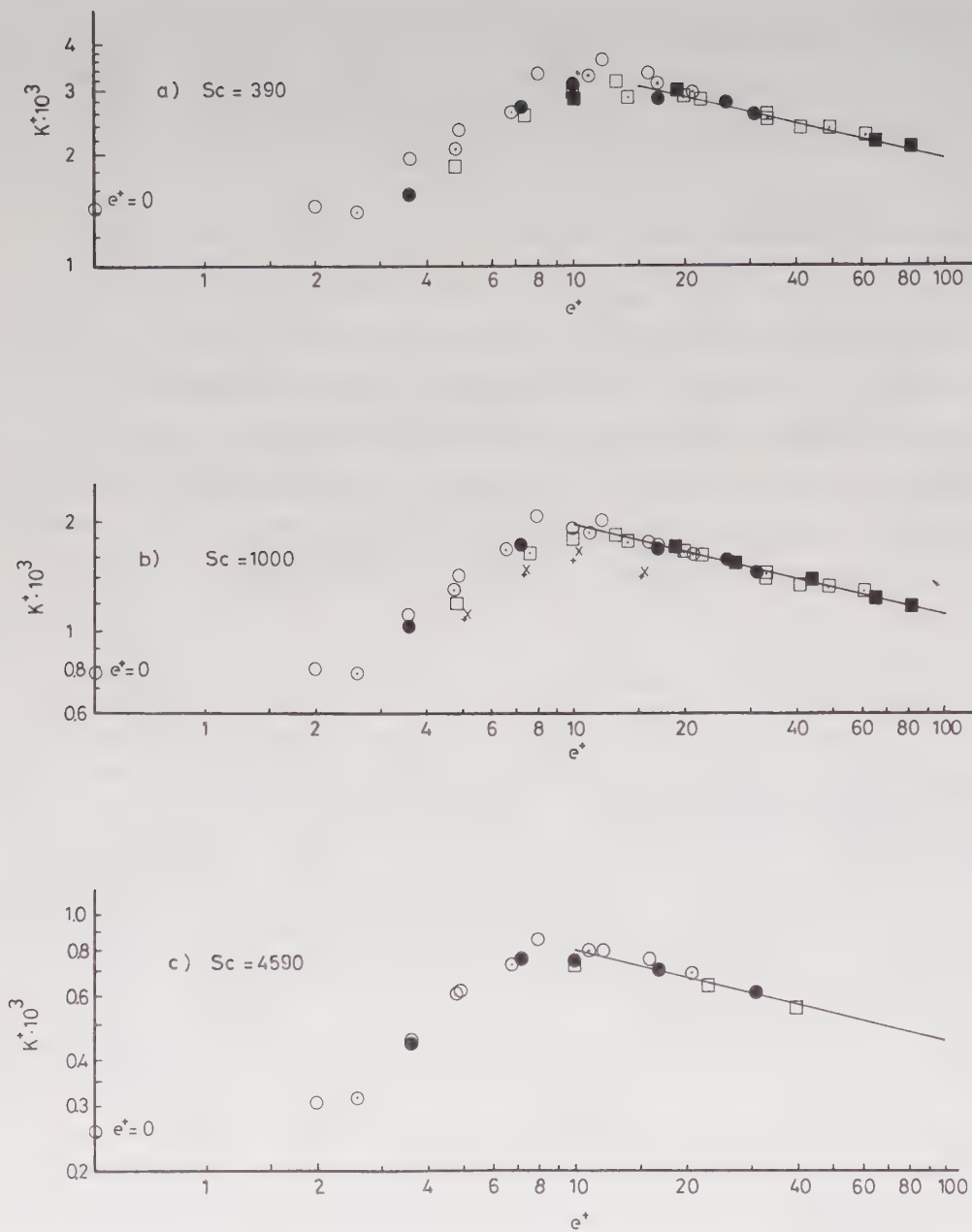


Fig. 22-1. k^+ vs e^+ , calculated from experimental data of Dawson and Trass (1972, Figs. 8-12)

Symbol $\bigcirc$ $\odot$ $\bullet$ $\square$ $\boxplus$ $\blacksquare$

$Re \cdot 10^{-3}$ 15 20 30 40 60 80

Constant roughness geometry with $\frac{e}{d_h} \cdot 10^3 = 2.2, 4.0, 5.2, 8.0, 11.0$ and 13.6

+ and x symbolise other roughness geometries with $\frac{e}{d_h} \cdot 10^3 = 4.0$

———— $k^+ = 0.197 \cdot Sc^{-0.59} \cdot (e^+)^{-0.237}$

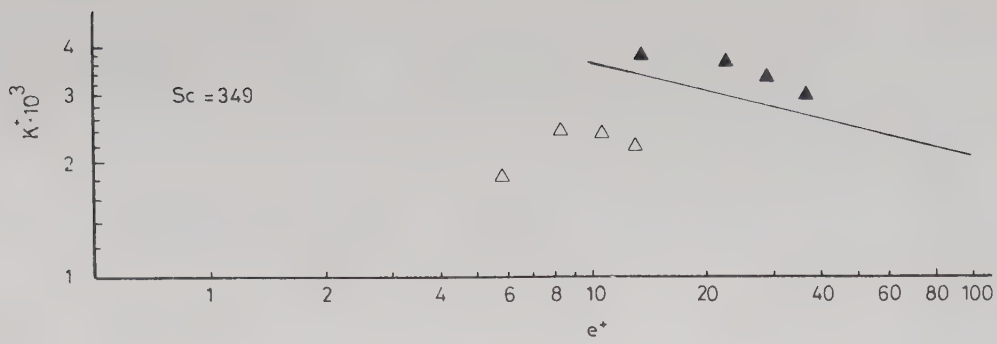


Fig. 22-2. k^+ vs e^+ , calculated from experimental data of Smith and Gowen (1965, Figs. 1 and 2). $3 \cdot 10^3 < Re < 6 \cdot 10^3$

$\triangle$ equivalent $\frac{e}{d_h} = 0.026$

$\blacktriangle$ equivalent $\frac{e}{d_h} = 0.06$

— $k^+ = 0.197 \cdot Sc^{-0.59} (e^+)^{-0.237}$

introduce errors into the calculations of the fully developed mass transfer coefficients which become too high. These errors will increase with decreasing Reynolds and Schmidt numbers. Probably this is the reason for the small dependence on the Reynold number of k^+ in Fig. 22-1. If so, the correlation (22-6) is fully confirmed by the results of Dawson and Trass.

At $Sc = 10^3$ Dawson and Trass also used two rough walls with each geometry differing slightly from the next. As seen in Fig. 22-1b, the mass transfer data from these geometries differ from the other, especially at $e^+ > 10$. Thus k^+ is influenced by the geometry of the roughness.

Smith and Gowen (1965) experimentally determined the heat transfer coefficient for two rough walls at $Pr = 349$. They used circular ducts which were internally roughened by threading. The two roughnesses were probably not geometrical scale consistent. They did not measure the heights of the roughness but determined the equivalent sand roughness diameters (equivalent $\frac{e}{d_h} = 0.026$ and 0.06) from momentum transfer data. Their heat transfer results are not very reliable as they did not consider the varying physical properties at different temperatures in the core region and at the wall. Further, their Figs. 1-3 are not consistent with each other (e.g. for $\frac{e}{d_h} = 0.06$ and $Re = 6000$). Nevertheless the results calculated from their figures 1 and 2 are shown in Fig. 22-2, and it is evident that k^+ reaches its maximum value at $e^+ \approx 10$, as for $Sc = 390$ in Fig. 22-1b. It is meaningless to compare further the results of Smith and Gowen with those of Dawson and Trass because of the inaccuracies of the former authors.

Discussion of Results

As seen in Fig. 22-1 there is one region, up to $e^+ \approx 10$, where k^+ increases with increasing e^+ . It appears that the value of e^+ for maximum k^+ decreases with increasing Schmidt number. At $Sc = 1000$ and $Sc = 4590$, k^+ has maximum for e^+ between 8 and 10 while at $Sc = 390$ the maximum is at e^+ between 11 and 13. Further k^+ is influenced of e^+ at lower e^+ the higher the Schmidt number. At $e^+ = 2$, k^+ is higher than at $e^+ = 0$ at $Sc = 4590$, which is not the case at $Sc = 390$. k^+ is relatively more affected by the roughness the higher the Schmidt number. The ratio $\frac{k^+_{max}}{k^+_{e=0}}$ is about 2.5, 2.7 and 3.3 at the Schmidt numbers 390, 1000 and 4590 respectively. The explanation for all these results is that wall roughness increases the whirl frequency

near the wall, by raising parts of the wall nearer the whirl generation region and perhaps by increasing the total whirl frequency. The higher the Schmidt number the more dependent is the mass transfer on the turbulent transport, so an increase in the turbulence level will increase the mass transfer rate relatively more the higher the Schmidt number.

The quantity of greatest interest is the dimensional mass transfer coefficient ($k = k^+ \cdot u_b \sqrt{\frac{f}{2}}$). The ratio $\frac{k}{k_{e=0}}$ at the same bulk velocity will thus be higher than $\frac{k^+}{k_{e=0}^+}$ by a factor $\sqrt{\frac{f}{f_{e=0}}}$. For $e^+ < 10$ this factor is usually below 1.2.

For values of e^+ higher than those which give a maximum for k^+ it seems that the correlation $k^+ = k(e^+)$ is a straight line in the log-log diagram. A least-squares linear regression is used to determine the straight line equation, and the result is shown in Table 22-1.

Sc	$e_{\min}^+$	Equation obtained
390	15	$k^+ = 5.82 \cdot 10^{-3} \cdot (e^+)^{-0.237}$
1000	10	$k^+ = 3.27 \cdot 10^{-3} \cdot (e^+)^{-0.237}$
4590	10	$k^+ = 1.39 \cdot 10^{-3} \cdot (e^+)^{-0.245}$

Table 22-1. Result of least-squares linear regression of data in Fig. 22-1. $e_{\min}^+$ is the minimum value of e^+ for which data have been used in the linear regression.

From Fig. 22-1 and Table 22-1 it is clear, without doubt, that k^+ is correlated to e^+ with the same power dependence for all Schmidt numbers. Furthermore the equations in Table 22-1 can be combined in a single equation

$$k^+ = 0.197 \cdot Sc^{-0.59} \cdot (e^+)^{-0.237} \quad (22-10)$$

The maximum deviation of Eq. (22-10) from the equations in Table 22-1 is 3% at $e^+ < 10^3$. Thus it is quite clear that for the roughness geometry used in the work of Dawson and Trass, k^+ is dependent on the Schmidt number and e^+ separately.

Dawson and Trass obtained the result, that the friction factors for all their rough walls were independent of e^+ at $e^+ > 10$. Their result is very well correlated (within 4%) with

$$f = 0.06 \left(\frac{e}{d_h}\right)^{0.38} \quad e^+ > 10 \quad (22-11)$$

Then Eqs. (22-10 and 11) together with the definitions of k^+ and e^+ (Eqs. 22-8 and 9) give

$$Sh = 0.052 \cdot \left(\frac{e}{d_h}\right)^{-0.092} \cdot Re^{0.763} \cdot Sc^{0.41} \quad \begin{cases} e^+ > 10 \text{ at } Sc > 1000 \\ e^+ > 15 \text{ at } Sc = 390 \end{cases} \quad (22-12)$$

Thus when the dimensionless height of the roughness has reached a certain value, which depends on the Schmidt number, the mass transfer coefficient at high Schmidt numbers has about the same power dependence on Re and Sc as in the case of smooth walls and low Schmidt numbers. The explanation is that at high Schmidt numbers and smooth walls most of the mass transfer resistance is in a wall region, where mainly stationary boundary layer flow prevails and hence $a \approx 0.88$ and $b \approx \frac{1}{3}$ in $Sh \approx Re^a \cdot Sc^b$. If the wall is now slightly roughened, the peaks will reach a level where the eddy frequency is much higher, and as the molecular mass diffusivity is low this will dramatically decrease the average mass transfer resistance in the wall region. As most of the total mass transfer resistance is in this region, the total mass transfer resistance will also dramatically decrease and thus the total mass transfer coefficient dramatically increases. The value of e^+ at which this will start is lower the higher the Schmidt number. For higher e^+ the mass transfer resistance in the wall region is small enough not to be totally dominating. Then if the mass transfer resistance in the wall region strongly decreases, the total mass transfer resistance need not decrease very much, so the increase in the total mass transfer coefficient is moderate. This is seen in Eq. (22-12) where increasing e^+ corresponds to increasing Reynolds number at constant $\frac{e}{d_h}$. A major part of the total mass transfer resistance is in a region where the mass is mainly transported by whirls as in the case at low Schmidt numbers with smooth walls. According to Higbie's penetration model $Sh \approx Sc^{0.5}$ for this type of turbulent transport, hence the exponent of the Schmidt number in Eq. (22-12) ought to be between 0.33 and 0.5, which is in fact the case.

In Eq. (22-12) the Sherwood number will decrease with increasing roughness height. Thus the rougher the wall the lower the mass transfer coefficient

at the same Reynolds number. However a ten-fold increase in the roughness height will only decrease the mass transfer coefficient by 20%. One possible explanation for this perhaps unexpected behaviour is, that when the peaks are in the whirl generation region (which starts at $y^+ \approx 5$ at smooth walls according to Corino and Brodkey (1969)), the mass transfer rate to the upper part of the wall surface will not increase with increasing roughness height. On the other hand so perhaps will the mass transfer rate at the lower parts of the wall decrease with increasing roughness height. Of the total mass transport to or from the wall probably only a small part is transferred at the lower parts of the wall, which is why the average mass transfer coefficient decreases very slightly with increasing roughness height.

Equation (22-12) is valid for the roughness geometry used by Dawson and Trass. The question is whether it is also valid for other roughness geometries. When looking at the results from other geometries by Dawson and Trass in Fig. 22-1b and by Smith and Gowen in Fig. 22-2, it seems that Eq. (22-12) can be used for those too. Of course the constant in Eq. (22-12) is different for different geometries and perhaps there will also be minor changes in the exponents of $\frac{e}{d_h}$ and Re . More experimental work however is needed to obtain definite knowledge about this.

Conclusion

For wall roughness which does not constitute any additional mass transfer resistance and at high Schmidt numbers ($Sc > 400$), the correlation for the mass transfer coefficient can be written $k^+ = k(Sc, e^+, \text{roughness geometry})$. At a certain value of e^+ the mass transfer coefficient begins to increase with increasing e^+ . The higher the Schmidt number the lower e^+ at which this begins. At e^+ in the order of 10, k^+ reaches its maximum value and the higher the Schmidt number the higher the increase in k^+ . The maximum increase in the dimensional mass transfer coefficient compared to the case with smooth walls is about three-fold and four-fold at $Sc = 390$ and $Sc = 4590$ respectively.

At $e^+ > 10$ the correlation for the mass transfer coefficient in dimensionless form is

$$Sh = a_1 \cdot \left(\frac{e}{d_h}\right)^{a_2} \cdot Re^{a_3} \cdot Sc^{0.41}$$

where a_1 and perhaps also a_2 and a_3 are influenced by the roughness geometry

$$a_1 \approx 5 \cdot 10^{-2}$$

$$a_2 \approx -0.092$$

$$a_3 \approx 0.763$$

23. ENTRANCE REGION

At the point at which mass transfer commences ($x = 0$) the concentration gradient at the wall and thus also the mass transfer coefficient are infinite. Unfortunately the wall area with infinite mass transfer coefficient is infinitesimal, so the transverse mass transport at the starting point is zero. Then the concentration gradient at the wall decreases with increasing x until a limit value is reached. Cases at high Schmidt numbers are considered in this chapter. For the case of heat transfer at turbulent air flow the reader is referred to Stephenson (1976). First the case of fully developed velocity profile and constant wall concentration is analysed. Then the case of simultaneously developing velocity and concentration profiles is briefly discussed.

Fully Developed Velocity Profile

The mass transfer in the entrance region is analysed under the following conditions

- fully developed velocity profile
- constant concentration before the entrance ($x < 0$)
- constant wall concentration (c_w)
- nonpermeable walls
- constant physical properties

At the commencement of the entrance region the concentration boundary layer is so thin, that within it at fully developed velocity profile the turbulent mass transport is negligible compared to the molecular one. Then under the above conditions, the equation of continuity for the solute will be

$$u \frac{\partial c}{\partial x} = D \left(\frac{\partial^2 c}{\partial y^2} + \frac{\partial^2 c}{\partial x^2} \right) \quad (23-1)$$

As the concentration boundary layer is so thin, rectangular coordinates can be used regardless of the wall curvature. Furthermore axial diffusion is negligible compared to the transverse diffusion, and very near the wall $u^+ = y^+$. Then Eq. (23-1) will be

$$y^+ \cdot \frac{\partial c^+}{\partial x^+} = \frac{1}{Sc} \frac{\partial^2 c^+}{\partial y^{+2}} \quad (23-2)$$

In this chapter c^+ is defined as $c^+ = \frac{c \cdot u^*}{(j_w)_\infty}$ where $(j_w)_\infty$ is the mass flux at infinite x . Equation (23-2) was proposed by Linton and Sherwood (1950).

By substituting

$$z = y^+ \left(\frac{Sc}{9x^+} \right)^{\frac{1}{3}} \quad (23-3)$$

Eq. (23-2) is transformed to an ordinary differential equation

$$\frac{d^2 c^+}{dz^2} + 3 z^2 \frac{dc^+}{dz} = 0 \quad (23-4)$$

$$\frac{dc^+}{dz} = a_1 \cdot e^{-z^3} \quad (23-5)$$

a_1 is a constant

With the boundary conditions

$$\text{B.C. 1} \quad c^+ = c_w^+ \quad \text{at} \quad z = 0$$

$$\text{B.C. 2} \quad c^+ = c_b^+ \quad \text{at} \quad z = \infty$$

Equation (23-5) gives

$$c_b^+ - c_w^+ = a_1 \int_0^\infty e^{-z^3} dz \quad (23-6)$$

$\int_0^\infty e^{-z^3} dz$ is identical with Gamma $\left(\frac{4}{3}\right) = 0.893$. The mass transfer coefficient is calculated from

$$k \cdot (c_b - c_w) = D \left(\frac{\partial c}{\partial y} \right)_w \quad (23-7)$$

Equations (23-3, 5, 6 and 7) give

$$k = \frac{u^* \cdot Sc^{-\frac{2}{3}} \cdot x^{+\frac{1}{3}}}{0.893 \cdot 9^{\frac{1}{3}}} \quad (23-8)$$

With $k^+ = \frac{k}{u^*}$ Eq. (23-8) gives

$$k^+ = 0.54 \cdot x^{+\frac{1}{3}} \cdot Sc^{-\frac{2}{3}} \quad (23-9)$$

Equation (23-9) is experimentally confirmed by Berger and Hau (1977) and Teng et al. (1979). Both used an electrochemical technique. Of course Eq. (23-9) is also applicable for laminar flow and then gives the well-known Leveque solution

$$Sh = 1.62 \left(\frac{d_h}{L} \cdot Re \cdot Sc \right)^{\frac{1}{3}}$$

The question is for how long axial distance Eq. (23-9) is valid. This equation is derived from Eq. (23-6) which is valid as long as the whole concentration boundary layer is within a wall region where the turbulent mass transport is negligible compared to the molecular one. Let the subscript "ℓ" denote the maximum value of respective quantity for which the turbulent mass transport is negligible. Then Eq. (23-6) and Eq. (23-9) are valid if

$$\int_0^{z_\ell} e^{-z^3} dz \approx \int_0^\infty e^{-z^3} dz$$

This equation is valid for all z_ℓ higher than a minimum value $(z_\ell)_{\min}$. The value of $(z_\ell)_{\min}$ depends on desired accuracy.

The eddy diffusivity model defines the mass eddy diffusivity (ϵ_q) according to

$$j = (D + \epsilon_q) \frac{dc}{dy} \quad (7-9)$$

Equation (23-9) is valid as long as $\frac{\epsilon_q}{D} \ll 1$. Let the limit be $\frac{\epsilon_q}{D} = a_1$, where a_1 is a constant. Empirical mass transfer data have shown that near the wall $\frac{\epsilon_q}{\nu} = a_2 \cdot y^{+a_3}$, where a_2 and a_3 are constants and a_3 is about three

Then Eq. (23-9) is valid up to $\frac{a_2 \cdot \nu \cdot y^{+a_3}}{D} = a_1$ or

$$y_\ell^+ = \left(\frac{a_1}{a_2 \cdot Sc} \right)^{\frac{1}{a_3}} \quad (23-10)$$

For this value of y_ℓ^+ , Eq. (23-3) gives the corresponding value of z

$$z_\ell = a_4 \cdot x^{+\frac{1}{3}} \cdot Sc^{\frac{1}{3}} - \frac{1}{a_3} \quad (23-11)$$

As the constant a_3 is equal to or near 3, z_ℓ is independent or only very slightly dependent on the Schmidt number. Hence with a high level of accuracy Eq. (23-11) can be written

$$z_\ell = a_4 \cdot x^+^{-\frac{1}{3}} \quad (23-12)$$

Thus z_ℓ will reach the value $(z_\ell)_{\min}$ at about the same value of x^+ for all high Schmidt numbers, which means that Eq. (23-9) is valid up to the same x^+ for all fluids with high Schmidt numbers. (At low Schmidt numbers the result perhaps will be slightly different because then Eq. (23-10) need not be valid in the whole concentration boundary layer.) Then according to the data of Teng et al. (1979, Fig. 4), Eq. (23-9) is valid to a value of x^+ of about 500. $x^+ = 500$ and $Re = 10^4$ give $\frac{x}{d_h} = 0.80$.

For a fully developed concentration boundary layer the correlation for k^+ can be written

$$k_\infty^+ = b \cdot Sc^{-\frac{2}{3}} \quad (23-13)$$

According to Shaw and Hanratty (1977a) the exponent of the Schmidt number at $Sc > 700$ is about -0.70 and not $-\frac{2}{3}$. However the choice of $-\frac{2}{3}$ makes it easier to compare Eq. (23-13) with Eq. (23-9), and Eq. (23-13) will give about the same result for both exponents. The value of x for which Eqs. (23-9 and 13) give the same result is denoted x_L and is obtained by combining these equations

$$x_L^+ = \left(\frac{0.54}{b}\right)^3 \quad (23-14)$$

By choosing $b = 0.063$, Eq. (23-13) deviates from the result of Shaw and Hanratty (1977a) by maximum 9% at $10^3 < Sc < 10^5$. For $x^+ = 500$ Eq. (23-9) gives $k^+ = 0.068 \cdot Sc^{-\frac{2}{3}}$, which is about 10% higher than k_∞^+ .

Now the question is from which value of x^+ has the local mass transfer coefficient reached the asymptotic value ($k = k_\infty$). By using the eddy diffusivity model the equation of continuity for the solute will be

$$y^+ \cdot \frac{\partial c^+}{\partial x^+} = \frac{D + \epsilon_q}{\nu} \frac{\partial^2 c^+}{\partial y^{+2}} \quad (23-15)$$

By defining z as before ($z = y^+ \cdot \left(\frac{Sc}{9x^+}\right)^{\frac{1}{3}}$) and assuming $\frac{\epsilon_q}{\nu} = a_2 \cdot y^{+3}$,

Eq. (23-15) can be written

$$z \cdot \left(\frac{Sc}{9x^+} \right)^{-\frac{1}{3}} \cdot \frac{\partial c^+}{\partial x^+} = \left(\frac{1}{Sc} + a_2 \cdot \frac{z^3 \cdot 9x^+}{Sc} \right) \cdot \left(\frac{Sc}{9x^+} \right)^{\frac{2}{3}} \cdot \frac{\partial^2 c^+}{\partial z^2} \quad (23-16)$$

$$z \cdot \frac{\partial c^+}{\partial x^+} = \left(\frac{1}{9x^+} + a_2 z^3 \right) \frac{\partial^2 c^+}{\partial z^2} \quad (23-17)$$

The boundary condtions are

B.C. 1	$c^+ = c_w^+$	at	$z = 0$
B.C. 2	$c^+ = c_b^+$	at	$z = \infty$
B.C. 3	$c^+ = c_b^+$	at	$x = 0$

Equation (23-17) and its boundary conditions give

$$c^+ = c(z, x^+, c_b^+, c_w^+) \quad (23-18)$$

Equation (23-7) gives

$$k = \frac{D \cdot u^* \cdot \left(\frac{\partial c^+}{\partial y^+} \right)_w}{v \cdot (c_b^+ - c_w^+)} \quad (23-19)$$

Of the parameters in the right side of Eq. (23-18), it is only z which depends on y^+ . Thus

$$\left(\frac{\partial c^+}{\partial y^+} \right)_w = \left(\frac{\partial c^+}{\partial z} \right)_w \cdot \left(\frac{\partial z}{\partial y^+} \right)_w \quad (23-20)$$

Equations (23-3, 19 and 20) give

$$k^+ = \frac{Sc^{-\frac{2}{3}}}{\frac{1}{(9x^+)^{\frac{2}{3}} \cdot (c_b^+ - c_w^+)}} \left(\frac{\partial c^+}{\partial z} \right)_w \quad (23-21)$$

From Eq. (23-18) one obtains that $\left(\frac{\partial c^+}{\partial z} \right)_w$ only depends on x^+ , c_w^+ and c_b^+

$$\left(\frac{\partial c^+}{\partial z}\right)_w = f_1(x^+, c_w^+, c_b^+) \quad (23-22)$$

By combining Eqs. (23-9 and 21) the result is for $x^+ < 500$

$$\left(\frac{\partial c^+}{\partial z}\right)_w = (c_b^+ - c_w^+) \cdot f_2(x^+) \quad x^+ < 500 \quad (23-23)$$

By combining Eqs. (23-13 and 21) the result is for high x^+

$$\left(\frac{\partial c^+}{\partial z}\right)_w = (c_b^+ - c_w^+) \cdot f_3(x^+) \quad \text{high } x^+ \quad (23-24)$$

Then probably Eq. (23-22) for all x^+ can be transformed to

$$\left(\frac{\partial c^+}{\partial z}\right)_w = (c_b^+ - c_w^+) \cdot f_4(x^+) \quad (23-25)$$

There is no mathematical proof of Eq. (23-25) but on physical grounds Eq. (23-25) seems to be realistic. Here it is assumed that Eq. (23-25) is valid for all x^+ . Then Eqs. (23-21 and 25) give

$$k^+ = Sc^{-\frac{2}{3}} \cdot f_5(x^+) \quad (23-26)$$

$$\frac{k^+}{k_\infty^+} = f_6(x^+) \quad (23-27)$$

Thus the mass transfer coefficient reaches its asymptotic value at about the same value of x^+ at all high Schmidt numbers.

According to the data of Linton and Sherwood (1950), k^+ has reached a value less than 1% higher than the asymptotic one at x^+ somewhere between 4000 and 8000. Assume that $k^+ = k_\infty^+$ at $x^+ = 5000$. Then for $x^+ > 5000$ the average mass transfer coefficient is calculated according to

$$\overline{k^+} = \frac{1}{x^+} \left[\int_0^{500} 0.54 (x^+)^{-\frac{1}{3}} \cdot Sc^{-\frac{2}{3}} \cdot dx^+ + \int_{500}^{5000} k^+ dx^+ + k_\infty^+ \cdot (x^+ - 5000) \right] \quad (23-28)$$

It is not possible to analytically obtain the dependence of x^+ on k^+ at

$500 < x^+ < 5000$. However the lower limit is $k^+ = 0.54 (x^+)^{-\frac{1}{3}} \cdot Sc^{-\frac{2}{3}}$ at $x^+ < x_L^+$ and $k^+ = k_\infty^+$ at $x^+ > x_L^+$ and the upper limit is, $k^+ = k(x^+)$ is

a linear function at $500 < x^+ < 5000$. Then Eq. (23-28) gives

$$\overline{k}_{\min}^+ = k_{\infty}^+ \left(1 + \frac{0.5 x_L^+}{x^+} \right) \quad (23-29)$$

Equations (23-13 and 28) give

$$\overline{k}_{\max}^+ = k_{\infty}^+ \left(1 + \frac{\frac{204}{b} - 2750}{x^+} \right) \quad (23-30)$$

If now $b = 0.063$, Eq. (23-14) gives $x_L^+ = 630$ and Eqs. (23-29 and 30) will be

$$1 + \frac{315}{x^+} < \frac{\overline{k}^+}{k_{\infty}^+} < 1 + \frac{489}{x^+} \quad (23-31)$$

It must be emphasised that the inequality (23-31) is not exact. It is built on the assumption that $k_{\infty}^+ \propto Sc^{-\frac{2}{3}}$, which is probably not exactly correct, and the constants in the inequality are strongly dependent on the value of "b". However the inequality (23-31) gives a good estimation of the deviation from the asymptotic value for the mass transfer coefficient.

Comparison with experimental data and other theoretical analyses

Shaw and Hanratty (1977a) based on experimental results claim that $x^+ > 10^4$ is needed in order that $\overline{k}^+ = k_{\infty}^+$. This is consistent with the inequality (23-31) which at $x^+ = 10^4$ gives $1.03 < \overline{k}^+/k_{\infty}^+ < 1.05$.

Berger and Hau (1977) experimentally obtained $\overline{k}^+/k_{\infty}^+ = 1.05$ at $\frac{x}{d_h} \approx 5$ and $\frac{x}{d_h} \approx 1$ at $Re = 10^4$ and $Re = 10^5$ respectively. At the same Reynolds numbers the inequality (23-31) gives $\overline{k}^+/k_{\infty}^+ = 1.05$ at $10 < \frac{x}{d_h} < 16$ and $1.3 < \frac{x}{d_h} < 2.0$ respectively. Thus it seems that the inequality (23-31) predicts too high average mass transfer coefficients. If so, this is probably because of the choice of a low value of "b".

Deissler (1955) and Sparrow et al. (1957) carried out theoretical analyses of heat transfer in the entrance region. However their results are not reliable. Mills (1962) shows that their results at $Pr = 0.73$ deviate somewhat from Mills' experimental data. For fully developed velocity profile and uniform wall heat flux, Deissler obtained that the heat transfer co-

efficient at $Pr = 10^3$ had reached its asymptotic value within 1% at $L^+ \approx 1200$. At $Pr = 0.73$, Deissler predicts that the heat transfer coefficient has reached its asymptotic value earlier at uniform wall temperature than at uniform heat flux. Presumably this will also be the case at $Pr = 10^3$. Then Deissler predicts that the heat transfer coefficient at fully developed velocity profile and uniform wall temperature has reached its asymptotic value before $L^+ = 1200$. The result of this work shows that this will not be the case until $L^+ > 4000$.

Sparrow et al. (1957) only considered a fully developed velocity profile and uniform wall heat flux. They obtained longer entrance lengths than Deissler did, but their entrance lengths were nearly independent of Reynolds number for all Prandtl numbers presented ($Pr \leq 100$). This contradicts the result for high Schmidt numbers in this work. The different boundary conditions (uniform wall heat flux vs. uniform wall temperature) cannot be a reason for the contradictory results. It should to be noted that at $Pr \approx 1$ the entrance length is almost independent of Reynolds number, according to both experimental results and other theoretical studies.

Undeveloped Velocity Profile

In technical applications the velocity profile is frequently not fully developed at the entrance of the mass transfer section. The mass transfer rate in the entrance region then depends on the turbulence intensity and velocity profile at the entrance. Henceforth in this chapter the entrance is defined as the position where the mass transfer commences.

Uniform velocity profile

When the velocity profile is uniform at the entrance there will be a radial flow in the positive y -direction to the centre region until the velocity profile is almost fully developed. This radial flow decreases the mass transfer coefficient, which is easy to understand in the case when $c_w = 0$, for example when the solute disappears by an electrode reaction at the wall. In this case the diffusional transport of the solute to the wall is counter-worked by the radial convection flow and thus the mass transport to the wall and the mass transfer coefficient decrease. In the case when the mass transport is from the wall towards the centre region, for example when the solute dissolves from the wall, the convective radial flow is in the same direction as the diffusional transport. However at the wall the radial con-

vective velocity is zero so the mass transport at the wall is by molecular diffusion only. Thus the mass transfer rate is directly proportional to the concentration gradient at the wall, which is decreased by the radial convective velocity. So even in this case the mass transport and thus also the mass transfer coefficient are decreased by the radial convective flow during the development of the velocity profile.

If there are no disturbances at the entrance, a laminar fluid boundary layer forms and extends downstream until its thickness is such that it becomes unstable and a transition to a turbulent boundary layer takes place. All the way the eddy diffusivity is lower than for the case with a fully developed velocity profile at the entrance. This means that for a given axial distance and all Schmidt numbers, the average mass transfer coefficient is higher when the velocity profile is fully developed at the entrance to the section than when the uniform entry velocity profile is without any flow disturbances.

Entry flow disturbances

In technical applications entry flow disturbances exist, for example a 90° sharp edge or a bend, which generate extra turbulence. This extra turbulence increases the mass transfer rate in the entry region. Only experimental investigations can determine this effect quantitatively, and for air heating such investigations have been made by, among others, Mills (1962) and Kakac et al. (1974). According to the results of Mills the inlet length to reach $\overline{Nu}/Nu_\infty = 1.03$ was 2-4 times longer for different entrance configurations compared with the fully developed velocity profile at the entrance. However from these results it is not possible to draw any conclusions for the case of mass transfer at high Schmidt numbers. This is because at high Schmidt numbers it is only the turbulence intensity very near the wall ($y^+ < 5$) which influences the mass transfer rate, while by air heating it is mainly the turbulence intensity outside this region which influences the heat transfer rate, because of the high thermal diffusivity. So before experimental investigations at high Schmidt numbers have been made, it is not possible to predict how much the entrance configuration influences the mass transfer rate at high Schmidt numbers in technical applications.

24. TURBULENT FLOW AT $Re < 10^4$

For flow in circular ducts, laminar flow prevails at $Re < 2100$. If the duct entrance is hydrodynamically well shaped and the walls are smooth, laminar flow can prevail at higher Reynolds number. In practice however, the conditions are often such that the transition from laminar to turbulent flow is just above $Re = 2100$. Within an interval the flow is labile, i.e. changeable laminar and turbulent, and within this region it is impossible to predict the mass and momentum transfer rates.

Theoretical Analysis

In practice turbulent flow always prevails at $Re > 4000$ for cylindrical tube flow. The majority of empirical correlations for the mass transfer coefficient are mostly said to be valid at $Re > 10^4$. A suitable starting point to examine the correlation for the mass transfer coefficient at $Re < 10^4$ is the result of Reichardt's analysis, which is valid for both turbulent and laminar flow. For fully developed flow with smooth nonpermeable walls Reichardt (1961) obtained

$$k = \frac{\frac{f}{2} \cdot u_b \cdot \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle \cdot \frac{u_{quot}}{c_{quot}}}{1 + \xi + (Sc_f - 1)b(Re, Sc) \cdot u_{quot} \cdot \sqrt{\frac{f}{2}}} \quad (24-1)$$

The term ξ is a measure of the deviation from one of the factor $\frac{j}{j_w} \cdot \frac{\tau_w}{\tau}$ over the cross sectional area. ξ increases with decreasing Reynolds and Schmidt numbers. However at high Schmidt numbers ($Sc > 500$) the sum $1 + \xi$ in the denominator is negligible compared to $(Sc_f - 1)b(Re, Sc)u_{quot}\sqrt{\frac{f}{2}}$. Further Reichardt found that at $Re \cdot Sc > 2500$ the factor $b(Re, Sc)$ was independent of Re , thus $b = b(Sc)$. Then Eq. (24-1) will be

$$k = \frac{\sqrt{\frac{f}{2}} \cdot u_b \cdot \left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle}{(Sc_f - 1)b(Sc) \cdot c_{quot}} \quad (24-2)$$

At high Schmidt numbers the whole concentration boundary layer is so thin that

- c_{quot} is very near one and independent of the Reynolds number
- $\left\langle \frac{\epsilon_q}{\epsilon_m} \right\rangle$, which is an average value of $\frac{\epsilon_q}{\epsilon_m}$ in the concentration boundary layer, is probably independent of the Reynolds number.

Thus for turbulent flow at high Schmidt numbers, $k \propto \sqrt{\frac{f}{2}} \cdot u_b$ for all Reynolds numbers, even below $Re = 10^4$. The physical reason for this is that at high Schmidt numbers the concentration boundary layer is so thin, that within it both the shear stress and the solute flux are constant even at low Reynolds numbers.

Experimental Data

Very few experimental works considering mass transfer at high Schmidt numbers and $3 \cdot 10^3 < Re < 10^4$ have been published. According to Son and Hanratty (1967), Shaw and Hanratty in 1964 obtained $k \propto \sqrt{\frac{f}{2}} \cdot u_b$ at $Sc = 2400$ and $7750 < Re < 50 \cdot 10^3$.

Mizushina et al. (1971) used a rectangular duct and obtained the result shown in Table 24-1.

Sc	$Re \cdot 10^{-3}$	Deviation from $k \propto \sqrt{\frac{f_{cyl}}{2}} \cdot u_b$ at the lower Re
2390	5.6 - 57	0%
3600	5.8 - 44	max. 5% below
5480	4.5 - 24	max. 4% below
9170	5.0 - 22	max. 10% below
15100	3.6 - 15	max. 7% below

Table 24-1. Deviation from $k \propto \sqrt{\frac{f_{cyl}}{2}} \cdot u_b$ according to Mizushina et al. (1971)

However Mizushina et al. did not measure the friction factor, but assumed that it would be the same as for circular ducts with the same Reynolds number. As will be seen in the next chapter this is not the case, but for the aspect ratio 10:1, which is used by Mizushina et al., at $Re < 5000$, the friction factor is less than that for a circular duct at the same Reynolds number ($f < f_{cyl}$). At $Re > 5000$ the opposite occurs ($f > f_{cyl}$). This may be the full explanation to the deviations shown in table 24-1.

Dawson and Trass (1972) measured the mass transfer coefficient in a square duct at Reynolds numbers down to 3000. They obtained a positive deviation from $k \propto \sqrt{\frac{f}{2}} \cdot u_b$ at low Reynolds numbers. However, as they admit themselves, their estimations of the friction factor were not accurate enough to confirm the positive deviation.

Derzansky and Gill (1974) calculated the mass transfer coefficient in a circular duct from reverse osmosis experimental data at $500 < Sc < 600$. At $Re = 2 \cdot 10^4$ and $Re = 10^4$ their results deviated only 1% and 3% respectively from the correlation obtained by Shaw and Hanratty (1977a).

$$Sh = 0.0889 \sqrt{\frac{f}{2}} \cdot Re \cdot Sc^{0.296} \quad (\text{Shaw-Hanratty}) \quad (24-3)$$

However at $Re < 5000$ their results did not agree with the correlation of Shaw and Hanratty, which is shown in Table 24-2.

Sc = 503				
Re	2122	3183	4244	5305
Sh (DG)	73	86	128	167
Sh (Eq. (24-3))	91	129	166	196
Sc = 516				
Re	2122	3183		
Sh (DG)		87		
Sh (Eq. (24-3))	91	130		
Sc = 529				
Re			4244	5130
Sh (DG)				172
Sh (Eq. (24-3))			169	199

Table 24-2. Comparisation of the result of Derzansky and Gill (1974, figures 5 and 6) (Sh (DG)) and Eq. (24-3).

From Table 24-2 it appears that the Sherwood number obtained from Eq. (24-3) agrees with the Sherwood number obtained by Derzansky and Gill at the nearest higher Reynolds number (dashed lines in Table 24-2). One explanation for this can be found from heat transfer data. Gnielinski (1976) showed that at $2300 < Re < 10^4$ and $0.7 < Pr < 100$ Eq. (24-4) correlated experimental data.

$$Nu = \frac{\frac{f}{2} \cdot (Re - 1000) \cdot Pr}{1 + 12.7 \sqrt{\frac{f}{2}} (Pr^{\frac{2}{3}} - 1)} \quad \left\{ \begin{array}{l} 2300 < Re < 10^4 \\ 0.7 < Pr < 100 \end{array} \right. \quad (24-4)$$

Thus the actual Reynolds number shall be reduced by 1000 when using the ordinary equation for calculating the Nusselt number at low Reynolds numbers. So also is the result of Derzansky and Gill at $Sc \approx 500$, as shown in Table 24-2.

According to Gnielinski (1976, figure 6) at $Pr = 700$, Eq. (24-4) underpredicts the Nusselt number, and the unreduced Reynolds number gives a good correlation for $Re > 5000$ but not for $Re < 5000$. Based on these observations it seems that $Sc = 700$ is not high enough and from Mizushima et al. (1971) it seems that $Sc = 2000$ is high enough to obtain $k \propto \sqrt{\frac{f}{2}} \cdot u_b$ over the whole turbulent Reynolds number range.

Conclusion

At $Sc > 2000$ the mass transfer coefficient can be calculated from Eq. (24-3) at all turbulent Reynolds numbers. At $Sc < 600$ and $3000 < Re < 10^4$ the mass transfer coefficient obtained from a correlation valid at $Re > 10^4$ shall be multiplied with $\frac{Re-1000}{Re}$ to obtain an approximation of the actual mass transfer coefficient. At $600 < Sc < 2000$ no clear conclusions can be drawn.

25. NONCIRCULAR DUCTS

Momentum Transfer

For duct geometries unsymmetrical of rotation the wall shear stress varies over the periphery. For example near corners the shear stress is relatively small. The varying wall shear stress causes secondary flows which aim at smoothing out the variations. The secondary flow is in a plane perpendicular to the axial direction and is shown schematically for a square duct in Fig. 25-1.

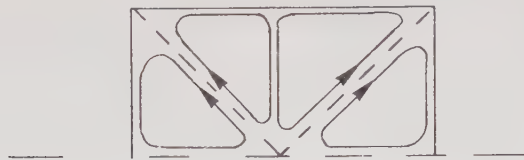


Fig. 25-1. Secondary flows in a square duct (schematically)

Brundrett and Burroughs (1967) experimentally examined the influence of the secondary flows on the wall shear stress in a square duct and the result is shown in Fig. 25-2.

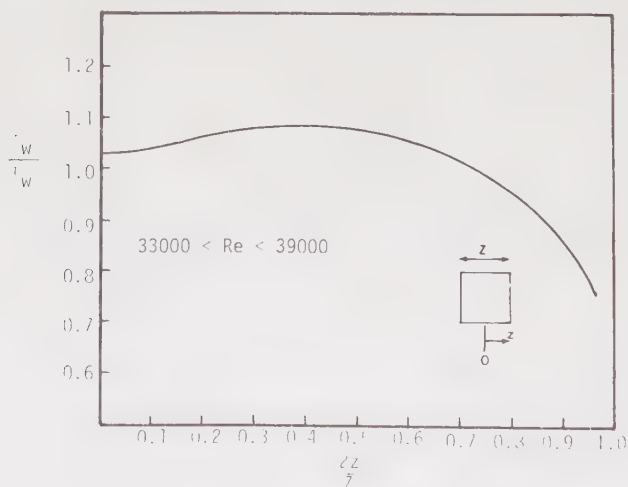


Fig. 25-2. The wall shear stress distribution in a square duct according to Brundrett and Burroughs (1967, Fig. 4)

From this figure it appears that at $Re \approx 3 \cdot 10^4$ the wall shear stress has a maximum at about half the distance from the corner to the side centre. The value of the wall shear stress is here about 9% higher than its average value. For more than 10% of the wall the wall shear stress is below 90% of its average value.

For rectangular ducts the importance of the secondary flows decreases with increasing aspect ratios, as then relatively less of the wall is near the corners. The wall shear stress near a corner will decrease with decreasing corner angle, and therefore the wall shear stress can vary very much over the periphery for for example triangular ducts.

For circular duct flow the wall shear stress is constant over the periphery, which causes the transition between laminar and turbulent flow to appear simultaneously over the whole cross section and then there is a sudden change in the friction factor. Gunn and Darling (1963) show that for ducts with varying wall shear stress, there is a gradual transition between laminar and turbulent flow. Turbulence first occurs in the open parts of the cross section, and the higher flow velocity turbulence prevails to an ever increasing extent of the cross section.

A force balance over a duct with constant cross section gives

$$\Delta P \cdot A = L \cdot \oint_S \tau_w dS \quad (25-1)$$

$$\Delta P = \frac{S \cdot L}{A} \cdot \overline{\tau}_w \quad (25-2)$$

$\overline{\tau}_w$ is the peripheral average value of the wall shear stress. The friction factor is defined according to Eq. (25-3)

$$f \equiv \frac{\overline{\tau}_w}{\frac{1}{2} \cdot \rho \cdot u_b^2} \quad (\text{def.}) \quad (25-3)$$

Equations (25-2 and 3) give

$$\Delta P = \frac{4L}{d_h} \cdot f \cdot \frac{1}{2} \cdot \rho \cdot u_b^2 \quad (25-4)$$

For smooth circular ducts and fully developed flow the friction factor depends on the Reynolds number only. Also for other duct geometries the friction factor depends on the Reynolds number only, but the correlations will not be the same for the different geometries if the hydraulic diameter is used for the characteristic length. Malák et al. (1975) and Mohandes and Knudsen (1979) have attempted to define a characteristic length other than the hydraulic diameter to obtain the same friction factor dependence on the Reynolds number for all duct geometries. How-

ever their work will not be considered in this study.

Probably the friction factor correlations will be the same for different duct geometries at high Reynolds numbers, as then the boundary layer for the velocity profile becomes very thin and hence the variations in the wall shear stress supposedly very small. This will occur at a higher Reynolds number the more the duct geometry deviates from the circular one. If this is the case, it is not possible with any characteristic length to use only one correlation for the friction factor for all duct geometries.

Mass Transfer

The secondary flows strongly decrease with decreasing distance to the wall. At high Schmidt numbers the concentration boundary layer is so thin that the secondary flows are negligible within it. Then the eddy diffusivity in the concentration boundary layer and thus also the mass transfer coefficient ought to be the same as for circular duct flow at the same wall shear stress. As the wall shear stress varies over the periphery, the mass transfer coefficient also varies. However very near the corners the turbulence in the concentration boundary layer is influenced by the corners, and thus the mass transfer coefficient here deviates from that at circular duct flow at the same wall shear stress. That part of the whole periphery where this deviation is valid is, however, so small that it does not affect the peripheral average mass transfer coefficient.

By fully developed turbulent circular duct flow with smooth walls and $Sc > 500$, the correlation for the mass transfer coefficient is

$$k = b \cdot \sqrt{\tau_w / \rho} \cdot f(Sc) \quad (25-5)$$

Thus the same equation ought to be valid also for other duct geometries, but it must be considered that the wall shear stress (τ_w) varies over the periphery.

$$\bar{k} = \frac{1}{S} \oint_S k \, dS \quad (25-6)$$

Equations (25-5 and 6) give

$$\bar{k} = b \cdot \sqrt{\tau_w / \rho} \cdot f(Sc) \quad (25-7)$$

The inequality of Schwarz gives

$$\bar{k} \leq b \cdot \sqrt{\tau_w / \rho} \cdot f(Sc)$$

Below rectangular, triangular and annular ducts are considered.

Rectangular Ducts

Brundrett and Burroughs (1967) and Hartnett et al. (1962) experimentally determined the friction factor for square ducts. Both obtained $f/f_{cyl} < 1$. The subscript "cyl" indicates the value of the quantity for circular duct flow at the same Reynolds number based on the hydraulic diameter.

The result of Hartnett et al. was

Re	$3 \cdot 10^3$	10^4	$> 2 \cdot 10^4$
f/f_{cyl}	≈ 0.88	≈ 0.97	≈ 1.0

The result of Brundrett and Burroughs was

$$f/f_{cyl} \approx 0.94 \quad \text{at} \quad 3 \cdot 10^4 < Re < 8 \cdot 10^4$$

The conclusion is that for square duct flow,

$$f/f_{cyl} < 1 \quad \text{and at} \quad Re > 10^4 \quad 0.94 < f/f_{cyl} < 1.$$

From Fig. 25-2 it is possible to determine the peripheral average value of the root of shear stress for square duct flow at $Re \approx 3 \cdot 10^4$. The result is $\sqrt{\tau_w} \approx 0.98 \cdot \sqrt{\tau_w}$. For that part of the wall which is between the side centre and half the distance to the corner, Fig. 25-2 gives $\sqrt{\tau_w} \approx 1.03 \cdot \sqrt{\tau_w}$. This means that at $Re \approx 3 \cdot 10^4$ and square duct flow the error will be 3% at maximum if Eq. (25-8) is used to calculate the average mass transfer coefficient for a whole or centre half side.

$$\bar{k} = b \cdot \sqrt{\tau_w / \rho} \cdot f(Sc) \quad (25-8)$$

b is the same constant as for circular duct flow.

For rectangular duct flow with high aspect ratio the result will be $f/f_{cy1} \geq 1$ except near the transition to laminar flow. Hartnett et al. (1962) obtained (at the aspect ratio of 10:1)

$$\begin{aligned} f/f_{cy1} &\leq 1 & Re &< 5 \cdot 10^3 \\ 1 < f/f_{cy1} &< 1.08 & 5 \cdot 10^3 < Re < 10^5 \\ f/f_{cy1} &\approx 1.0 & Re &> 10^5 \end{aligned}$$

f/f_{cy1} had a maximum at about $Re = 2 \cdot 10^4$.

Mizushina et al. (1971) experimentally determined the mass transfer coefficient at high Schmidt numbers for turbulent flow in a rectangular duct with an aspect ratio of 10:1. An electrochemical method was used and the measurements of mass transfer rates were made by a cathode which covered 40% of the width of one of the long sides. The centres of the cathode and the long side coincided. They assumed that $f = f_{cy1}$ and from this calculated

$$k_{Miz}^+ = k/u_b \cdot \sqrt{f_{cy1}/2} \quad (25-9)$$

However k^+ which only depends on the Schmidt number at high Schmidt numbers is obtained from

$$k^+(Sc) = k/u_b \cdot \sqrt{\frac{f}{2}} \quad (25-10)$$

Equations (25-9 and 10) give

$$k_{Miz}^+ = k^+(Sc) \cdot \sqrt{\frac{f}{f_{cy1}}} \quad (25-11)$$

Mizushina et al. from Eq. (25-9) obtained at all Schmidt numbers ($600 < Sc < 15000$) the following

- k_{Miz}^+ increases with increasing Reynolds number at $Re < \approx 2 \cdot 10^4$
- k_{Miz}^+ decreases with increasing Reynolds number at $\approx 2 \cdot 10^4 < Re < 8 \cdot 10^4$

The variations of k_{Miz}^+ were always less than 10%. These results agree with Eq. (25-11) and the result of Hartnett et al. which predict a maximum for $\frac{f}{f_{cy1}}$ at $Re \approx 2 \cdot 10^4$.

Conclusion

If the same correlations for calculating the friction factor and mass transfer coefficient are used for square and rectangular duct flow as for circular duct flow, the resulting error in the mass transfer coefficient will be less than 5% at $Sc > 500$ and $Re > 10^4$.

Triangular Ducts

Carlson and Irvine (1961) experimentally examined the friction factor in isosceles triangular duct flow. At $2 \cdot 10^3 < Re < 3 \cdot 10^4$ their results were

apex angle ($^\circ$)	4.01	7.96	12.0	28.3	38.8	60
f/f_{cyl}	0.78	0.83	0.86	0.91	0.95	0.95

It is evident that the friction factor decreases with decreasing apex angle. The wall shear stress varies more in triangular duct flow than in rectangular duct flow and it is not possible to calculate a local or an average mass transfer coefficient without knowing the wall shear stress distribution for the actual wall area. This has been done by Aly et al. (1978) for equilateral triangular duct flow, i.e. at 60° apex angle. The examined Reynolds number range was $53 \cdot 10^3 < Re < 107 \cdot 10^3$ and from figure 6 in the above mentioned paper is obtained

$$\sqrt{\tau_w} \approx 0.99 \cdot \sqrt{\tau_w} \quad 53 < Re \cdot 10^{-3} < 107 \quad (25-12)$$

Aly et al. further obtain $0.935 < f/f_{cyl} < 0.95$, which agrees with the results of Carlson and Irvine (1961).

Equations (25-7 and 12) together with the value of f/f_{cyl} give that for fully developed equilateral triangular duct flow at $Sc > 500$ and $Re > 50 \cdot 10^3$ the correlation for the mass transfer coefficient will be

$$\bar{k} \approx 0.97 \cdot k_{cyl} \quad (25-13)$$

Thus if the correlations for circular duct flow is used the error will be about 3%.

Concentric Cylindrical Annular Ducts

A concentric cylindrical annular duct is symmetrical through rotation, so the shear stress does not vary over the outer or inner periphery. However, the velocity profile is unsymmetrical, which causes difficulties in determining the positions of maximum velocity (r_v) and zero shear stress (r_τ) in turbulent flow. For a long time it was assumed that r_v and r_τ coincided but Kjellström and Hedberg (1966) showed theoretically that this need not be the case. The expression for the shear stress is

$$\tau_{yx} = \tau_{yx}^{(m)} + \tau_{yx}^{(t)} = -\mu \frac{\partial u}{\partial y} + \rho \overline{u_y' \cdot u_x'} \quad (25-14)$$

The prime denotes the deviation from the time-smoothed value. At the position of maximum velocity the term $\tau_{yx}^{(m)}$ is zero, but if the velocity profile is unsymmetrical, there is no reason to suppose that $\tau_{yx}^{(t)}$ is also zero. By accurate measurements Rehme (1974, 1975a and 1975b) showed that the position for zero shear stress was nearer the inner wall than the position for maximum velocity. The distance between these increased with increasing unsymmetrical velocity profile, i.e. with decreasing ratio of the quotient "radius of the inner cylinder (r_i)/radius of the outer cylinder (r_o)". However

$$r_i/r_o > 0.5 \quad \text{gives} \quad r_\tau \approx r_v$$

According to Rehme (1974) the position of zero shear stress at $Re \approx 10^5$ can be calculated from

$$\frac{r_\tau - r_i}{r_o - r_\tau} = \left(\frac{r_i}{r_o}\right)^{0.386} \quad Re \approx 10^5 \quad (25-15)$$

The more symmetrical the velocity profile the less the dependence of r_τ on Reynolds number. From the data of Rehme it seems that for $r_i/r_o > 0.1$, Eq. (25-15) is valid for the whole Reynolds number range examined ($2 \cdot 10^4 < Re < 2 \cdot 10^5$). Then Eq. (25-15) gives the result shown in Table 25-1.

$\frac{r_i}{r_o}$	0.5	0.6	0.7	0.8	0.9	0.95
$\frac{r_\tau - r_i}{r_o - r_i}$	0.43	0.45	0.47	0.48	0.49	0.50

Table 25-1. The position of zero shear stress in fully developed flow in concentric cylindrical annular ducts with smooth walls.
 $2 \cdot 10^4 < Re < 2 \cdot 10^5$

Table 25-1 ought to be valid down to $Re \approx 10^4$. Near the transition point between laminar and turbulent flow the flow is more complicated. In the outer region ($r_\tau < r < r_o$) both the cross sectional area and the average fluid velocity are greater than in the inner region ($r_i < r < r_\tau$). This causes the turbulence to occur first in the outer region. The turbulence in the outer region influences the flow in the inner region but first at a higher flow velocity the inner region itself generates turbulence. The position of zero shear stress for this transition range is not known.

A force balance gives

$$\Delta P \cdot \pi \cdot (r_\tau^2 - r_i^2) = \tau_i \cdot 2\pi r_i \cdot \Delta L \quad (25-16)$$

$$\Delta P \cdot \pi \cdot (r_o^2 - r_\tau^2) = \tau_o \cdot 2\pi r_o \cdot \Delta L \quad (25-17)$$

where τ_i and τ_o are the wall shear stresses at the inner and outer walls of the annulus respectively.

Equations (25-15, 16 and 17) together with Eq. (25-4) give the result shown in Table 25-2.

r_i/r_o	0.5	0.6	0.7	0.8	0.9	0.95
$\tau_i / f \cdot \frac{1}{2} \cdot \rho u_b^2$	1.05	1.04	1.02	1.01	1.01	1.00
$\tau_o / f \cdot \frac{1}{2} \cdot \rho u_b^2$	0.97	0.98	0.98	0.99	0.99	1.00

Table 25-2. The inner and outer wall shear stresses in fully developed flow in concentric cylindrical annular ducts with smooth walls. $Re > 10^4$

Rehme (1974) examined concentric cylindrical annular ducts with $r_i/r_o = 0.02, 0.04$ and 0.1 at $2 \cdot 10^4 < Re < 2 \cdot 10^5$ and obtained $0.95 < f/f_{cyl} < 1.05$. The higher the value of r_i/r_o the higher the value of f/f_{cyl} and for $r_i/r_o = 0.1$ he obtained $f/f_{cyl} > 1$ over the whole Reynolds number range examined. At high values of r_i/r_o the flow conditions will be the same as for flow between plane parallel plates or in rectangular ducts with high aspect ratios. According to Hartnett et al. (1962) the maximum of f/f_{cyl} is 1.08 for rectangular duct flow with aspect ratio of 10:1. Then for concentric cylindrical annular duct flow Eq. (25-18) is probably valid

$$1 < \frac{f}{f_{cyl}} < 1.08 \quad \left\{ \begin{array}{l} Re > 10^4 \\ \frac{r_i}{r_o} > 0.1 \end{array} \right. \quad (25-18)$$

For the whole Reynolds number range examined Rehme obtained that the dimensionless velocity near the outer wall, and for $r_i/r_o = 0.1$ also near the inner wall, was the same function of the dimensionless distance from the wall as in circular duct flow. This means that the eddy diffusivity near the wall follows the same equation ($\frac{\epsilon_m}{\nu} = f(y^+)$) in concentric cylindrical annular duct flow as in circular duct flow. Thus at high Schmidt numbers ($Sc > 500$), $k^+ = f(Sc)$ will be the same in concentric cylindrical annular duct flow as in circular duct flow at $Re > 10^4$ and $r_i/r_o > 0.1$. The dimensional mass transfer coefficient (k), however, will not be exactly the same for the outer and inner walls, depending on the different shear stresses according to table 25-2.

When combining Table 25-2, Eq. (25-18) and $k = b \cdot \sqrt{\frac{\tau_w}{\rho}} \cdot f(Sc)$ one obtains for fully developed flow in concentric cylindrical annular ducts with smooth walls

$$0.98 \cdot k_{cyl} < k < 1.07 \cdot k_{cyl} \quad \left\{ \begin{array}{l} Re > 10^4 \\ r_i/r_o > 0.5 \\ Sc > 500 \end{array} \right. \quad (25-19)$$

Conclusion

If the same correlations for the friction factor and the mass transfer coefficient are used as for circular duct flow, the resulting error in the mass transfer coefficient will be less than 7% at $Re > 10^4$, $r_i/r_o > 0.5$ and $Sc > 500$.

26. VARIABLE PHYSICAL PROPERTIES

The physical properties which influence the mass transfer rate are the solute-solvent diffusivity and the viscosity and density of the solution. These properties might depend on static pressure, temperature, solute concentration and the shear stress. It is possible that the physical properties also depend on the distribution between the turbulent and molecular momentum transfer rates. However within the concentration boundary layer at high Schmidt numbers the turbulent momentum transfer rate is negligible compared to the molecular one.

In turbulent flow at high Schmidt numbers the concentration boundary layer is so thin that within it the shear stress is practically constant. Here only liquids are considered and for these the physical properties do not vary with small pressure variations. Furthermore, only isothermal conditions are considered. The result is that in the concentration boundary layer at high Schmidt numbers the physical properties vary only with the solute concentration.

Momentum Transfer

The wall shear stress for a specified duct depends on the bulk velocity and density and the viscosity of the fluid. Here a theoretical analysis is carried out for the case when the viscosity is higher at the wall than in the bulk region.

The thickness of the concentration boundary layer is δ_q and within it the viscosity is higher than outside. We now look at the limit, when the viscosity is infinite in the whole concentration boundary layer. This means that this layer is stagnant and as $\delta_q \ll d_h$ the cross sectional area available to flow passage decreases from $S \cdot d_h/4$ to $S(d_h/4 - \delta_q)$. The percentage decrease in the open area will then be

$$\frac{400 \cdot \delta_q^+}{Re \cdot \sqrt{\frac{f}{2}}}$$

δ_q^+ is in the order of 1 at $Sc > 10^3$. If it is assumed that $\delta_q^+ = 3$ (from Deissler (1955, figure 2)), the percentage decrease in the cross sectional area available to flow passage will be 5, 2 and 0.5% at $Re = 3 \cdot 10^3$, 10^4 and $5 \cdot 10^4$ respectively. Further if it is assumed that the eddy diffusivity

is the same as if the stagnant layer is a solid wall, the increase in the wall shear stress will be 10, 4 and 1% respectively at the above Reynolds numbers compared to the case with constant physical properties. In practice the viscosity in the concentration boundary layer is not infinite, so the increase in the wall shear stress should be much less than predicted above. Thus at high Schmidt numbers the wall shear stress is not noticeably influenced by concentration dependent physical properties. The bulk properties will be used in calculating the shear stress.

Some high molecular weight polymers (drag reducing polymers) decrease the shear stress by reducing the whirl production in the generation region. However, this phenomenon will not be discussed in this study.

Mass Transfer

There was a demand for experimental works in order to discover the correlation for the mass transfer coefficient at turbulent flow and constant physical properties. Still more difficulties arise when the physical properties are concentration dependent. Mostly no reliable diffusivity versus concentration data exist. Probstein et al. (1979) show that even for such an often used solution as bovine serum albumin (BSA) in water, there are big differences between diffusivity data published by different authors. Consequently at present there are no well proven correlations for the mass transfer coefficient at turbulent flow and concentration dependent physical properties. Here a theoretical analysis will be done.

Theoretical analysis

Reichardt's (1961) analysis included variable physical properties. At high Schmidt numbers ($Sc > 500$) and with the assumption $\epsilon_m = \epsilon_q$ in the concentration boundary layer in this analysis gives

$$k = \frac{u^*}{\int_0^{u_c^+} Sc \frac{D}{D+\epsilon_m} du^+} \quad (26-1)$$

The Schmidt number inside the integral varies in the concentration boundary layer, but it can be placed outside the integral if a weighted average value, $\bar{Sc}$, is used. As the quotient $\frac{D}{D+\epsilon_m}$ in the integral strongly de-

creases with increasing distance from the wall, the value of $\overline{Sc}$ is near Sc_w . Then Eq. (26-1) can be written

$$k = \frac{u^*}{\overline{Sc} \cdot \int_0^{u_c^+} \frac{du^+}{1 + \frac{\epsilon_m}{D}}} \quad (26-2)$$

The problem is to find the correlation $\frac{\epsilon_m}{D} = f(u^+)$ or $\frac{\epsilon_m}{\nu} \cdot Sc = f(u^+)$. According to Hanna and Sandall (1978) Goldman in 1954 proposed that u^+ and $\frac{\epsilon_m}{\nu}$ at variable physical properties were correlated to y^{++} in the same way as to y^+ at constant physical properties. y^{++} is defined according to Eq. (26-3).

$$y^{++} = \int_0^y \frac{u^*}{\nu} dy \quad (26-3)$$

However there is no theoretical or experimental proof for the proposal that $\frac{\epsilon_m}{\nu}$ is a function of only y^{++} , but if this is the case, Eq. (26-2) can be written

$$k = \frac{u^*}{\overline{Sc} \int_0^{y_c^{++}} \frac{dy^{++}}{1 + f(y^{++}) \cdot Sc}} \quad (26-4)$$

In the concentration boundary layer $\epsilon_m \ll \nu$, therefore, $u^+ = y^{++}$. It is not possible to generally integrate Eq. (26-4) as the Schmidt number varies with the concentration and thus also with y^{++} . However it is possible to determine the maximum and minimum value of k by using Sc_w and Sc_b respectively for the Schmidt number inside the integral. With Goldman's proposal and $\frac{\epsilon_m}{\nu} \propto y^{++3}$, Eq. (26-4) gives

$$\frac{Sc_b}{Sc} < \frac{k}{k_u} < \left(\frac{Sc_w}{Sc_b}\right)^{\frac{1}{3}} \cdot \frac{Sc_b}{Sc} \quad (26-5)$$

where k_u is the mass transfer coefficient at constant physical properties equal to the actual bulk values.

Equation (26-1) can be written

$$k = \frac{u^*}{y_c^+ \int_0^{y_c^+} \frac{du^+}{\frac{1}{Sc} + \frac{\epsilon_m}{v}}} \quad (26-6)$$

Equation (26-6) and the proposal of Goldmann with $\frac{\epsilon_m}{v} \propto y^{++3}$ give

$$\left(\frac{Sc_b}{Sc_w}\right)^{\frac{2}{3}} < \frac{k}{k_u} < 1 \quad (26-7)$$

The Inequalities (26-5 and 7) give

$$\left(\frac{Sc_b}{Sc_w}\right)^{\frac{2}{3}} < \frac{k}{k_u} < \left(\frac{Sc_b}{Sc_w}\right)^{\frac{2}{3}} \cdot \frac{Sc_w}{Sc} \quad (26-8)$$

As earlier mentioned $\overline{Sc}$ is probably only slightly less than Sc_w . The Inequality (26-8) and Eq. (23-13) then give

$$\frac{k}{k_u} \approx \left(\frac{Sc_b}{Sc_w}\right)^{\frac{2}{3}} \quad (26-9)$$

$$k \approx b \cdot u_b \cdot \sqrt{\frac{f}{2}} \cdot Sc_w^{-\frac{2}{3}} \quad (26-10)$$

Once again it ought to be pointed out that Eq. (26-9) is obtained from the proposal of Goldmann, which has not been verified.

Equation (26-10) predicts that it is mainly the wall values of the physical properties which determine the mass transfer coefficient. Of the bulk properties it is only the viscosity which slightly influences the mass transfer coefficient via the friction factor. The insensibility of the mass transfer coefficient to the bulk properties has been verified by Goldsmith (1971). by ultrafiltration of polyethylene glycol (mean molecular weight 15500).

Heat transfer analogy

There are many experimental works which consider variable physical proper-

ties in heat transfer. However the conditions are here different from those at concentration dependent physical properties. For liquids the thermal diffusivity varies only slightly with the temperature. For example for water the thermal diffusivity decreases by only 22% while the viscosity increases 500% by a temperature decrease from 100°C to 0°C. For gases the Prandtl number is low and is besides almost independent of the temperature. Thus by heat transfer it is sufficient to consider the temperature dependence of the viscosity. A common correlation for the Nusselt number which considers variable viscosity is

$$Nu = Nu_u \left(\frac{\mu_b}{\mu_w} \right)^n \quad (26-11)$$

$$n = 0.11 \quad \text{heating}$$

$$n = 0.25 \quad \text{cooling}$$

However this form is not suitable for mass transfer as then the mass diffusivity varies at least as much as the viscosity with the concentration.

Sleicher and Rouse (1975) examined published works about turbulent heat transfer in circular ducts with variable viscosity. From these they selected the most reliable data and so obtained 18 combinations between the Nusselt number and the Reynolds and Prandtl numbers. Only heating was considered and the Prandtl number range was $3 < Pr_b < 75$. These 18 combinations were very well correlated with

$$Nu_b = 5 + Re_f^a \cdot Pr_w^b \quad (26-12)$$

$$a = 0.88 - \frac{0.24}{4 + Pr_w}$$

$$b = \frac{1}{3} + 0.5 \exp(-0.6 \cdot Pr_w)$$

The subscript f refers to the "film temperature" $T_f = \frac{T_b + T_w}{2}$. Equation (26-12) is the same as the Sleicher and Rouse expression for constant physical properties (Eq. 12-5) with the film Reynolds number and wall Schmidt number. To test Eq. (26-12) it is applied to heating and cooling of water, which is shown in Tables 26-1 and 2.

Data of physical properties are from VDI-Wärmeatlas (1977).

a)	$T_b = 50^{\circ}\text{C}$	$Pr_b = 3.54$	$\nu_b = 0.551 \cdot 10^{-6} \text{ m}^2/\text{s}$
	$T_w (^{\circ}\text{C})$	Pr_w	$\nu_f \cdot 10^6 \text{ m}^2/\text{s}$
			$\frac{h}{h_u}$ (according to Eq. (26-12))
	60	2.96	0.510
	70	2.53	0.471
	80	2.20	0.438
	90	1.94	0.409

b)	$T_b = 20^{\circ}\text{C}$	$Pr_b = 6.94$	$\nu_b = 1.00 \cdot 10^{-6} \text{ m}^2/\text{s}$
	$T_w (^{\circ}\text{C})$	Pr_w	$\nu_f \cdot 10^6 \text{ m}^2/\text{s}$
			$\frac{h}{h_u}$ (according to Eq. (26-12))
	30	5.39	0.890
	40	4.30	0.800
	50	3.54	0.720
	60	2.96	0.656
	70	2.53	0.600
	80	2.20	0.551
	90	1.94	0.510

Table 26-1. Equation (26-12) applied to heating of water.

$$u_b \cdot d_h = 27.55 \cdot 10^{-3} \text{ m}^2/\text{s}$$

a)	$T_b = 50^{\circ}\text{C}$	$Pr_b = 3.54$	$\nu_b = 0.551 \cdot 10^{-6} \text{ m}^2/\text{s}$
	$T_w (^{\circ}\text{C})$	Pr_w	$\nu_f \cdot 10^6 (\text{m}^2/\text{s})$
			$\frac{h}{h_u}$ (according to Eq. (26-12))
	40	4.30	0.600
	30	5.39	0.656
	20	6.94	0.72
	10	9.28	0.800

b)	$T_b = 20^{\circ}\text{C}$	$Pr_b = 6.94$	$\nu_b = 1.00 \cdot 10^{-6} \text{ m}^2/\text{s}$
	$T_w (^{\circ}\text{C})$	Pr_w	$\nu_f \cdot 10^6 (\text{m}^2/\text{s})$
			$\frac{h}{h_u}$ (according to Eq. (26-12))
	10	9.28	1.133
	0	13.0	1.300

Table 26-2. Equation (26-12) applied to cooling of water.

$$u_b \cdot d_h = 27.55 \cdot 10^{-3} \text{ m}^2/\text{s}$$

In Table 26-1a the heat transfer coefficient decreases with increasing wall to bulk temperature difference by heating, and in Table 26-2 the heat transfer coefficient increases with increasing temperature difference by cooling. These results are not consistent with experimental data, so Eq. (26-12) cannot be recommended neither for heating nor cooling and of course nor for mass transfer data.

Conclusion

It is not possible to use heat transfer data to obtain correlations for the mass transfer coefficient at variable physical properties. When $Sc_w > Sc_b$, Eqs. (26-10 and 13) are recommended at $Sc > 500$.

$$k \approx b \cdot u_b \cdot \sqrt{\frac{f}{2}} \cdot Sc_w^{-\frac{2}{3}} \quad (26-10)$$

$$Sh_w \approx b \cdot \sqrt{\frac{f}{2}} \cdot Re_w \cdot Sc_w^{\frac{2}{3}} \quad (26-13)$$

The friction factor is calculated from the bulk physical properties.

27. PERMEABLE WALLS

With permeable walls there will be convective momentum and mass transport between the wall and the bulk region. The convective flow affects the mass transfer coefficient. To analytically study this, the film theory has been used (e.g. Bird et al. (1960, Chapter 21). However the film theory is not realistic, so here the eddy diffusivity model is used to obtain a correlation for the mass transfer coefficient at convective flow through the wall.

The Eddy Diffusivity

The question is whether the eddy diffusivity is influenced by convective flow through the wall. Kinney and Sparrow (1970) performed an extensive analysis of the influence by permeable walls on momentum and heat transfer. They assumed that the eddy diffusivity increased by wall suction at relatively high permeate flows. Based on experiments, Brosh and Winograd (1974) claimed that the eddy diffusivity decreased by wall suction. Probably the eddy diffusivity is influenced by transverse convective flow, but to what extent is not fully known. However at small transverse velocities this influence ought to be negligible.

Theoretical Analysis

The requirements are

- fully developed turbulent flow
- constant physical properties
- a high Schmidt number so the concentration boundary layer is thin enough to let the transverse velocity and solute flux (v_w and j_w respectively) be constant within it.
- that the eddy diffusivity is not influenced by the transverse velocity

Figure 27-1 shows the conditions near the wall.

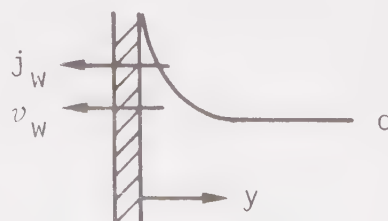


Fig. 27-1. The concentration profile at wall suction and nonpermeable solute

A mass balance in the concentration boundary layer gives

$$j = c \cdot v + (D + \epsilon_q) \frac{dc}{dy} \quad (27-1)$$

The concentration boundary layer is thin enough for j and v to be constant within it, j_w and v_w respectively. In dimensionless form Eq. (27-1) will be

$$1 = c^+ \cdot v_w^+ + \left(\frac{1}{Sc} + \frac{\epsilon_q}{v} \right) \frac{dc^+}{dy^+} \quad (27-2)$$

$$\frac{d}{dy^+} \left(c^+ \cdot \exp \left(\int_0^{y^+} \frac{v_w^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} dy^+ \right) \right) = \frac{1}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} \exp \left(\int_0^{y^+} \frac{v_w^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} dy^+ \right) \quad (27-3)$$

$$\frac{d}{dy^+} \left(c^+ \cdot \exp \left(\int_0^{y^+} \frac{v_w^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} dy^+ \right) \right) = \frac{d}{dy^+} \left(\frac{1}{v_w^+} \exp \left(\int_0^{y^+} \frac{v_w^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} dy^+ \right) \right) \quad (27-4)$$

$$c^+ = \frac{1}{v_w^+} + c_1 \cdot \exp \left(- \int_0^{y^+} \frac{v_w^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} dy^+ \right) \quad (27-5)$$

where c_1 is a constant.

The boundary condition $c^+ = c_w^+$ at $y^+ = 0$ gives

$$c_1 = c_w^+ - \frac{1}{v_w^+}$$

This inserted in Eq. (27-5) gives

$$c_b^+ - c_w^+ = \left(c_w^+ - \frac{1}{v_w^+} \right) \left[\exp \left(-v_w^+ \int_0^\infty \frac{dy^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} \right) - 1 \right] \quad (27-6)$$

At high Schmidt numbers Eq. (27-7) is valid

$$\int_0^\infty \frac{dy^+}{\frac{1}{Sc} + \frac{\epsilon_q}{v}} = \frac{u^*}{k(v_w^+ = 0)} \quad (27-7)$$

where $k(v_w=0)$ is the mass transfer coefficient under the same conditions except for nonpermeable walls.

Equations (27-6 and 7) give

$$c_b^+ - c_w^+ = (c_w^+ - \frac{1}{v_w^+}) \left[\exp(-\frac{v_w^+ \cdot u^*}{k(v_w=0)}) - 1 \right] \quad (27-8)$$

The mass transfer coefficient is defined according to Eq. (27-9)

$$j_w \equiv k(c_b - c_w) + v_w \cdot c_w \quad (\text{def.}) \quad (27-9)$$

Equation (27-9) gives

$$c_b^+ - c_w^+ = (1 - c_w^+ \cdot v_w^+) \cdot \frac{u^*}{k} \quad (27-10)$$

Equations (27-8 and 10) give

$$-\frac{v_w^+ \cdot u^*}{k} = \exp(-\frac{v_w^+ \cdot u^*}{k(v_w=0)}) - 1 \quad (27-11)$$

$$\frac{v_w}{k} = 1 - e^{-\frac{v_w}{k(v_w=0)}} \quad (27-12)$$

Equation (27-12) is exactly the same result as given by the film theory. The film theory is the special case when $\epsilon_q = 0$ in this analysis.

At the wall ϵ_q is zero and Eq. (27-1) then gives

$$j_w = c_w \cdot v_w + D \left. \frac{dc}{dy} \right|_w \quad (27-13)$$

Equations (27-9 and 13) give that for both permeable and nonpermeable walls, Eq. (27-14) is valid

$$k = \frac{D \left. \frac{dc}{dy} \right|_w}{c_b - c_w} \quad (27-14)$$

Thus the mass transfer coefficient is a measure of the concentration gradient near the wall. According to Eq. (27-12) the mass transfer coefficient increases with increasing wall suction and decreases with increasing wall injection. Physically this means that the concentration gradient at the wall increases with increasing wall suction and decreases with increasing wall injection.

Experimental Results

Mizushina et al. (1972) used an electrochemical method to determine the influence of the transverse velocity on the mass transfer coefficient. The conditions were

Circular duct flow

$$7900 < Re < 21000$$

$$750 < Sc < 2100$$

$$-0.9 \cdot 10^{-3} < u_w^+ < 0.5 \cdot 10^{-3}$$

Their results are in excellent agreement with Eq. (27-12). Thus in the range $-0.9 < u_w^+ \cdot 10^3 < 0.5$ the eddy diffusivity is independent of the transverse velocity.

Thomas (1973) used a dynamic membrane on a carbon tube support and a celluloseacetate membrane to study the influence of the transverse velocity on the mass transfer coefficient in reverse osmosis. The dynamic membrane gave a high suction rate and the celluloseacetate membrane a low wall suction rate and Thomas obtained a much higher mass transfer coefficient for the former than for the latter. However the increase in the mass transfer coefficient for the dynamic membrane is mainly because the carbon tube has a rough wall, so from Thomas' results it is not possible to find out the influence of the transverse velocity on the mass transfer coefficient.

Some Aspects concerning the Importance of Mass and Momentum Transfer in Reverse Osmosis and Ultrafiltration

Permeate flux

In reverse osmosis and ultrafiltration the solvent permeates through the wall (membrane) by a pressure difference. The mass flux of the solute

through the membrane can be written

$$j_w = v_w \cdot c_p \quad (27-15)$$

c_p = solute concentration in the permeate.

Combination of Eqs. (27-9 and 15) gives

$$-\frac{v_w}{k} = -\frac{c_b - c_w}{c_p - c_w} \quad (27-16)$$

Equations (27-12 and 16) give

$$v_w = k(v_w=0) \cdot \ln \frac{c_w - c_p}{c_b - c_p} \quad (27-17)$$

Equation (27-17) shows the connection between the permeate flux and the wall concentration.

In ultrafiltration c_p is often negligible compared with c_b and c_w and there is an upper limit for c_w where gelling occurs. Then Eq. (27-17) can be used to predict maximum possible permeate flux. This will be directly proportional to the mass transfer coefficient ($k(v_w=0)$) according to the same equation. Usually in these cases the physical properties of the fluid vary with the concentration. The mass transfer coefficient may then be calculated as in chapter 26, but if the density of the solution varies considerably the transverse velocity v is not constant in the concentration boundary layer. Then Eq. (27-12) and thus also Eq. (27-17) are not reliable in the vicinity of $c_w = c_g$. Henceforth it is assumed that the density variation is small enough for the above equations to be valid.

In reverse osmosis the solution-diffusion model gives

$$v_w = Q(\Delta P - \Delta\pi) \quad (27-18)$$

$$j_w = B (c_w - c_p) \quad (27-19)$$

where Q and B are constant at moderate pressures and at specified membrane and solution. $\Delta\pi$ is the osmotic pressure difference $\pi_w - \pi_p$. For

not too concentrated solutions, the osmotic pressure of the solution is directly proportional to the solute concentration. Let γ denote the proportionality factor. Then Eq. (27-19) gives

$$j_w = B \cdot \gamma \cdot \Delta\pi \quad (27-20)$$

Equations (27-17 and 18) give

$$\exp(v_w/k(v_w=0)) = \frac{\Delta P - \frac{v_w}{Q}}{\pi_b - \pi_p} \quad (27-21)$$

Equation (27-21) is also valid for ultrafiltration as long as $v_w/k(v_w=0)$ is not too large. Equation (27-17) must give $c_w < c_g$.

For small $v_w/k(v_w=0)$, $\exp(v_w/k(v_w=0)) \approx 1 + v_w/k(v_w=0)$. For $v_w/k(v_w=0) < 0.2$ the error will be less than 2%. Then Eq. (27-21) gives

$$v_w = \frac{Q(\Delta P - (\pi_b - \pi_p))}{1 + \frac{Q(\pi_b - \pi_p)}{k(v_w=0)}} \quad (\text{small } \frac{v_w}{k(v_w=0)}) \quad (27-22)$$

Equations (27-15, 17 and 19) give

$$c_b - c_p = \frac{c_b}{1 + \frac{B}{v_w} \exp(v_w/k(v_w=0))} \quad (27-23)$$

Equations (27-22 and 23) give

$$v_w = \frac{Q \left(\Delta P - \frac{\pi_b}{1 + \frac{B}{v_w} \exp(v_w/k(v_w=0))} \right)}{1 + \frac{Q \cdot \pi_b}{k(v_w=0) \left(1 + \frac{B}{v_w} \cdot \exp(v_w/k(v_w=0)) \right)}} \quad (27-24)$$

It is impossible to get v_w in a simple explicit form in the general case, but when the membrane is impermeable for the solute, i.e. $B=0$, Eq. (27-24) will be

$$v_w = \frac{Q(\Delta P - \pi_b)}{1 + \frac{Q \cdot \pi_b}{k(v_w=0)}} \quad \left(\begin{array}{l} \text{small } \frac{v_w}{k(v_w=0)} \\ B = 0 \end{array} \right) \quad (27-25)$$

Thus there will be a linear relationship between the permeate flux (v_w) and ΔP with a slope little less than Q and an intersection of the abscissa at $\Delta P = \pi_b$.

In reverse osmosis "tight" membranes are used to stop small molecules. The permeate flux is usually below $14 \cdot 10^{-6} \text{ m/s}$ ($50 \text{ l/m}^2\text{h}$). If the flow is turbulent the bulk velocity is usually above 0.5 m/s . At room temperature the Schmidt numbers are about 500 and 2000 for salt and sugar solutions respectively. Then Eq. (21-8) gives

$$k(v_w=0) > 13 \cdot 10^{-6} \text{ m/s} \quad (\text{salt and sugar solutions, turbulent flow})$$

Then usually $v_w < k(v_w=0)$

A typical value of the permeability constant of a reverse osmosis membrane is $Q = 2 \cdot 10^{-12} \text{ m}^2\text{s/kg}$. Figure 27-2 shows the permeate flux calculated from Eq. (27-21) for a salt or sugar solution. It appears that at low bulk osmotic pressure (0.1 MPa) there is a negligible dependence of the permeate flux on the mass transfer coefficient. At a bulk osmotic pressure of 1.5 MPa the permeate flux can be increased by more than 20% by increasing the mass transfer coefficient from $13 \cdot 10^{-6} \text{ m/s}$ to $52 \cdot 10^{-6} \text{ m/s}$.

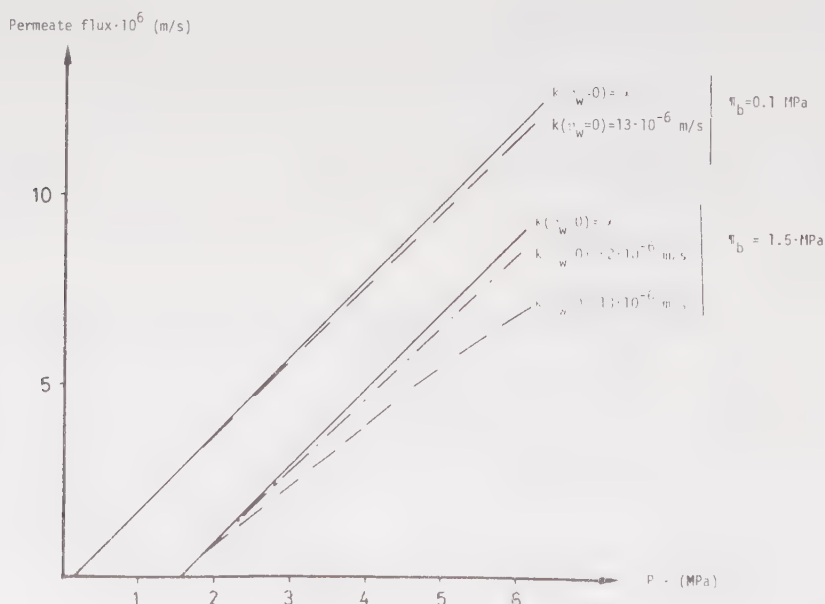


Fig. 27-2. Reverse Osmosis of a salt or sugar solution. From Eq. (27-21) with $\pi_p = 0$ and $Q = 2 \cdot 10^{-12} \text{ m}^2\text{s/kg}$.

However in practice there are no pure binary solutions, but usually impurities are present which foul the membrane. Then an increase in the bulk velocity may increase the permeate flux relatively more than Fig. 27-2 shows by decreasing the wall fouling layer. However, in any case Eq. (27-21) and Fig. 27-2 give the maximum obtainable permeate flux.

Solute concentration in the permeate

Equations (27-12 and 16) give

$$e^{-\frac{u_w}{k(u_w=0)}} = \frac{c_p - c_b}{c_p - c_w} \quad (27-26)$$

If the solution-diffusion model is valid, one obtains from Eqs. (27-15, 19 and 26)

$$e^{-\frac{u_w}{k(u_w=0)}} = \frac{B}{u_w} \left(\frac{c_b}{c_p} - 1 \right) \quad (27-27)$$

$$c_p = \frac{c_b}{1 + \frac{u_w}{B \cdot \exp(u_w/k(u_w=0))}} \quad (27-28)$$

If c_b , $k(u_w=0)$ and B are constant and u_w varies, for example by increasing the static pressure difference, one obtains for the variation of c_p

$$\frac{\partial c_p}{\partial u_w} = \frac{c_b \cdot B \cdot \exp(u_w/k(u_w=0))}{\left[1 + \frac{u_w}{B \cdot \exp(u_w/k(u_w=0))} \right]^2} \left(\frac{u_w}{k(u_w=0)} - 1 \right) \quad (27-29)$$

Equation (27-29) gives

$$\frac{u_w}{k(u_w=0)} < 1 \rightarrow \frac{\partial c_p}{\partial u_w} < 0$$

$$\frac{u_w}{k(u_w=0)} > 1 \rightarrow \frac{\partial c_p}{\partial u_w} > 0$$

Thus if the permeate flux (u_w) is increased by increasing the static press-

ure, c_p first decreases, until u_w is equal to $k(u_w=0)$ and then increases. This means that at $u_w > k(u_w=0)$, an increase in the static pressure will increase the solute wall concentration (c_w) relatively more than the permeate flux.

Energy consumption in ultrafiltration

Figure 27-3 shows a typical ultrafiltration system. The feed flow, per-

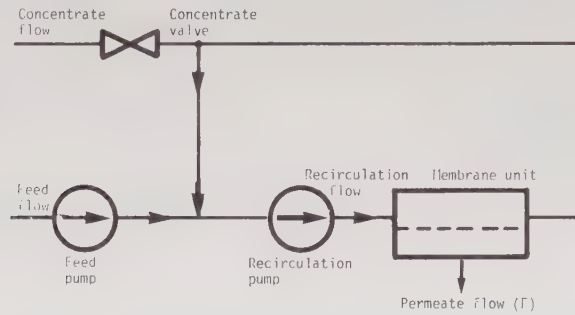


Fig. 27-3. A typical ultrafiltration system.

meate flow and concentrate flow are specified. The membrane unit consists of N channels in parallel each with hydraulic diameter d_h and active membrane length L . The entire periphery of the active membrane length is covered by membrane. Then the total membrane area (I) is

$$I = \pi \cdot d_h \cdot L \cdot N \quad (27-30)$$

Turbulent flow prevails with the bulk velocity u_b . Then the pressure drop over the membrane is

$$\Delta P = \frac{4L}{d_h} \cdot f \cdot \frac{1}{2} \cdot \rho \cdot u_b^2 = \frac{4L}{d_h} \cdot \rho \cdot u^{*2} \quad (27-31)$$

Then recirculation flow is $N \cdot u_b \cdot \frac{\pi d_h^2}{4}$. Then the effect (E) of the motor to the recirculation pump is

$$E = \frac{1}{\eta \cdot \beta} \cdot \frac{4L}{d_h} \cdot \rho \cdot u^{*2} \cdot N \cdot u_b \cdot \frac{\pi d_h^2}{4} \quad (27-32)$$

η = the efficiency of the motor + pump

β = the fraction of the total pressure drop which is over the length of active membrane

$$E = \frac{I \cdot \rho \cdot u_b \cdot u^{*2}}{\eta \cdot \beta} \quad (27-33)$$

The mass transfer coefficient $k(u_w=0)$ is

$$k(u_w=0) = b \cdot u^* \cdot f(Sc) \quad (27-34)$$

Equations (27-17 and 34) and the condition $c_p = 0$ give

$$u_w = b \cdot u^* \cdot f(Sc) \cdot \ln \frac{c_w}{c_b} \quad (27-35)$$

The permeate flow (F) then will be

$$F = I \cdot b \cdot u^* \cdot f(Sc) \cdot \ln \frac{c_w}{c_b} \quad (27-36)$$

The energy consumption in the recirculation loop per unit volume permeate is E/F . Equations (27-33 and 36) give

$$\frac{E}{F} = \frac{\rho \cdot u_b \cdot u^*}{\eta \cdot b \cdot f(Sc) \cdot \ln \frac{c_w}{c_b}} \quad (27-37)$$

The energy consumption is low if the bulk velocity is low. However, at low bulk velocities the mass transfer coefficient is low, so a large membrane area is needed to produce a specified permeate flow. To minimize the total cost a high bulk velocity and consequently a high energy consumption must be chosen. The operation pressure is often high enough to let the permeate flux (u_w) be limited by gelling at the wall. Then if the total membrane area and the permeate flow are constant also u^* must be constant according to Eq. (27-36).

The definition of u^* ($u^* = u_b \cdot \sqrt{\frac{f}{2}}$) and the Blasius expression for the friction factor ($f = 0.0791 \cdot Re^{-1/4}$) give

$$u_b = 6.3 \cdot (u^*)^{8/7} \cdot \left(\frac{d_h}{v}\right)^{1/7} \quad (27-38)$$

Equations (27-37 and 38) and $c_w = c_g$ give

$$\frac{E}{F} = \frac{6.3 \cdot \rho \cdot (u^*)^{15/7} \cdot d_h^{1/7}}{\eta \cdot b \cdot v^{1/7} \cdot f(Sc) \cdot \ln \frac{c_g}{c_b}} \quad (27-39)$$

Thus at constant membrane area, permeate flow, wall solute concentration and nonpermeable solute, the energy consumption per unit volume permeate has a very weak dependence on the hydraulic diameter of the channel ($E/F \propto d_h^{-1/7}$). If the hydraulic diameter is doubled the energy consumption increases 10%. Much more important is the pressure drop efficiency, i.e. that part of the total pressure drop in the recirculation loop which is over the active membrane area.

NOTATION

A	cross section area	m^2
a	different definitions in different chapters	
B	solute permeability constant	m/s
b	different definitions in different chapters	
c	solute concentration	kg/m^3
$c^+ = cu^*/j_w$	in chapter 23, $c^+ = cu^*/(j_w)_\infty$	
c_p	in Table 1-1, heat capacity/unit mass at constant pressure	J/kg K
D	mass diffusivity	m^2/s
$D_e = D + \epsilon_q$		m^2/s
$d_h = 4 A/s$	hydraulic diameter	m
E	effect	W
e	height of roughness	m
$e^+ = e \cdot u^*/\nu$		
F	permeate flow	m^3/s
$f = \frac{1}{S} \int_S \tau_w dS / \frac{1}{2} \rho \cdot u_b^2$	fannings friction factor	
g	gravitational force per unit mass	m/s^2
h	heat transfer coefficient	m/s
I	wall area	m^2
j	solute flux in direction to the wall with respect to stationary axis	kg/m^2s
k	mass transfer coefficient	m/s
$k^+ = k/u^*$		
L	axial length	m
m	mass	kg
N	number	
n	eddy frequency	s^{-1}
$n^+ = n \cdot \nu / u^{*2}$		
P	static pressure	N/m^2

Q	permeability constant	$m^2 \cdot s/kg$
r	radial distance from the centre	m
$r^+ = r \cdot u^*/\nu$		
S	length of periphery	m
s	renewed fraction per unit time	s^{-1}
T	temperature	K
t	time	s
u	axial velocity	m/s
$u^* = \sqrt{\tau_w/\rho}$	friction velocity	m/s
$u^+ = u/u^*$		
v	transverse velocity	m/s
ν	transverse velocity	m/s
$\nu^+ = \nu/u^*$		
x	axial distance	m
$x^+ = x \cdot u^*/\nu$		
y	transverse distance from the wall	m
$y^+ = y \cdot u^*/\nu$		
$y^{++} = \int_0^y (u^*/\nu) dy$		
$z = y^+ \cdot (Sc/9x^+)^{\frac{1}{3}}$		
$\alpha = \lambda/\rho c_p$	thermal diffusivity	m^2/s
β	fraction of the pressure drop which occurs over the membran	
$\gamma = c/\pi$		s^2/m^2
Δ	difference	
δ	thickness of a film or boundary layer	m
$\delta^+ = \delta \cdot u^*/\nu$		
ε	eddy diffusivity	m^2/s
η	efficiency of pump + motor	
θ	contact time	s

$$\theta^+ = \theta u^{*2} / \nu$$

λ	thermal conductivity	J/msk
ν	momentum diffusivity (Kinematic viscosity)	m^2/s
$\nu_e = \nu + \epsilon_m$		m^2/s
ξ	defined in chapter 11	
π	3.14 or the osmotic pressure	N/m^2
ρ	density	kg/m^3
τ	shear stress	N/m^2
ψ	dummy	

Subscripts

b	bulk value, $\psi_b = \frac{1}{A \cdot u_b} \int_A \psi dA$ $\psi \neq u$, $u_b = \frac{1}{A} \int_A u dA$
c	centerline vlaue
cr	core region value
cyl	the quantity at circular duct flow at the same Schmidt number and Reynolds number based on the hydraulic diameter
e	u_e^+ defined in Eq. (22-2)
f	quantity evaluated at the temperature $(T_b + T_w)/2$
i	quantity evaluated at the inner area
ir	intermediate region value
is	at unsteady condition
L	x_L is defined just above Eq. (23-14)
ℓ	maximum value of the quantity for which ϵ_q is negligible
Mi_z	$k_{Mi_z}^+$ is defined in Eq. (25-9)
m	momentum transfer quantity
min	minimum value
o	quantity evaluated at the outer area
p	permeate value
pump	pump quantity
q	mass transfer quantity

quot	$\psi_{\text{quot}} = (\psi_b - \psi_w)/(\psi_c - \psi_w)$
s	at stationary condition
u	quantity evaluated at constant physical properties, equal to the actual bulk values
v	quantity evaluated at maximum velocity
w	wall value
wr	wall region value
x	in the x-direction
y	in the y-direction
δ	quantity evaluated at the distance δ from the wall
θ	value at the end of the contact time
τ	quantity evaluated at zero shear stress
ψ	dummy
0	initial condition
∞	quantity at fully developed velocity and concentration profiles

Superscripts

(m)	molecular part
(t)	turbulent part
'	deviation from time-smoothed value
+	dimensionless

Overlines

$\overline{}$	average value
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Brackets

$\langle \rangle$	average value
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Dimensionless Groups

$$Nu = h \cdot d_h / \alpha$$

$$Pr = \nu / \alpha$$

$$Re = d_h \cdot u_b / \nu$$

$$Sc = \nu / D$$

$$Sc_e = \nu_e / D_e$$

$$Sh = k \cdot d_h / D$$

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